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OLIY MATEMATIKA

MASALALAR TO`PLAMI

I QISM

Institutning barcha bakalavriat ta'lim yo`halishlari uchun

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Annotatsiya. Ushbu masalalar to`plami institutning barcha bakalavriat ta’lim yo`halishlari uchun mo`ljallangan bo`lib, unga “Oliy matematika” fanidan aniqlovchilar, matritsalar, chiziqli tenglamalar sistemasi va ularni yechish usullari, tekislikda va fazodagi analitik geometriya elementlari haqida qisqacha tushuncha kiritilgan. Har bir mavzuga oid masalalar namunaviy yechimlari, mustaqil ishlash uchun masalalar kiritilgan.

Masalalar to`plami “Matematika” kafedrasi majlisida muhokama qilingan va nashrga tavsiya etilgan.

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“IQTISOD-MOLIYA” nashriyoti, 2005

1. IKKINCHI, UCHINCHI TARTIBLI ANIQLOVCHILARNI HISOBLASH

a_{11}, a_{12}, a_{21}, a_{22} haqiqiy sonlar berilgan bo'lsin ikkinchi tartibli determinant (yoki aniqlovchi) deb, $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ kabi belgilanuvchi va

$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$ tenglik bilan aniqlanuvchi songa aytiladi

1. a) $\begin{vmatrix} 3 & -4 \\ 2 & 5 \end{vmatrix} = 3\cdot 5-(-4)\cdot 2 = 15+8 = 23$

   b) $\begin{vmatrix} \sqrt{a} & -1 \\ a & \sqrt{a} \end{vmatrix} = \sqrt{a}\cdot\sqrt{a} - (-1)\cdot a = a+a = 2a$

Mustaqil yechish uchun misollar:

Quyidagi ikkinchi tartibli determinantlarni hisoblang:

2. $\begin{vmatrix} -7 & 6 \\ 5 & -4 \end{vmatrix}$

3. $\begin{vmatrix} 10 & -5 \\ 9 & -8 \end{vmatrix}$

4. $\begin{vmatrix} \sqrt{a}+\sqrt{b} & \sqrt{a}-\sqrt{b} \\ \sqrt{a}-\sqrt{b} & \sqrt{a}+\sqrt{b} \end{vmatrix}$

5. $\begin{vmatrix} \sin 1^0 & \sin 89^0 \\ -\cos 1^0 & \cos 89^0 \end{vmatrix}$

6. $\begin{vmatrix} (x+y)/x & 2x/(x-y) \\ (y-x)/(x^2-y^2) & (y-x)/(x^2-y^2) \end{vmatrix}$

7. $\begin{vmatrix} \sin^2 a & \cos^2 a \\ \sin^2 b & \cos^2 b \end{vmatrix}$

8. Tenglamani yeching:

a) $\begin{vmatrix} x & 3 \\ 1 & 2x \end{vmatrix} + 3\begin{vmatrix} 0,(4) & x \\ 1 & 3 \end{vmatrix} = 0$

b) $(0,6)\cdot(25/9)\begin{vmatrix} x & 3 \\ 4 & x \end{vmatrix} = (27/125)^3$

9. Tengsizliklarni yeching:

a) $\begin{vmatrix} x & 1 \\ -4 & x \end{vmatrix} \le \begin{vmatrix} 5 & 2 \\ 1 & x \end{vmatrix}$,

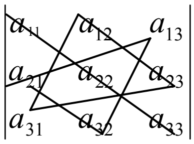
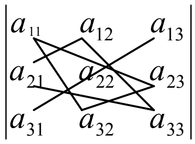
b) $1/\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix} < 1/3$

Berilgan a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33} haqiqiy sonlardan tuzilgan $a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$ yig'indiga teng va

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$ kabi berilgan songa uchinchi tartibli determinant deb ataladi.

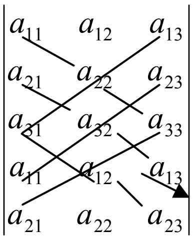
Uchinchi tartibli determinantlarni uchburchaklar usulida, Sarryus usulida hamda biror satr yoki ustun elementlari bo'yicha yoyib hisoblash mumkin.

1. Uchburchaklar usuli:

(+)  (-) 

$$a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

2. Sarryus usuli:



$$=a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

3. Birinchi ustun elementlari bo'yicha yoyib hisoblash:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

10. Uchinchi tartibli determinantlarni uchrurchaklar usuli, Sarryus usuli hamda biror ixtiyoriy satr yoki ustun elementlari bo'yicha yoyib hisoblang:

a) $$\begin{vmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 4 & -1 & -5 \end{vmatrix} = 1\cdot(-3)\cdot(-5) + 1\cdot 1\cdot 4 + 2\cdot(-1)\cdot 1 - 1\cdot(-3)\cdot 4 - 1\cdot 2\cdot(-5) - 1\cdot(-1)\cdot 1 =$$

$$= 15 + 4 - 2 + 12 + 10 + 1 = 40$$

b) $$\begin{vmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 4 & -1 & -5 \\ 1 & 1 & 1 \\ 2 & -3 & 1 \end{vmatrix} = 15 - 2 + 4 + 12 + 1 + 10 = 40$$

c) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 4 & -1 & -5 \end{vmatrix} = 1\begin{vmatrix} -3 & 1 \\ -1 & -5 \end{vmatrix}$ - $1\begin{vmatrix} 2 & 1 \\ 4 & -5 \end{vmatrix} + 1\begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} = 16 + 14 + 10 = 40$

Mustaqil yechish uchun misollar:

Quyidagi uchinchi tartibli determinantlarni qulay usulda hisoblang:

11. $\begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

12. $\begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix}$

13. $\begin{vmatrix} 5 & 3 & 2 \\ -1 & 2 & 4 \\ 7 & 3 & 6 \end{vmatrix}$

14. $\begin{vmatrix} 1 & 2 & 3 \\ 8 & 1 & 4 \\ 2 & 1 & 1 \end{vmatrix}$

15. $\begin{vmatrix} 3 & -1 & -2 \\ 1 & 2 & 5 \\ -4 & 1 & 6 \end{vmatrix}$

16. $\begin{vmatrix} 1 & 2 & -1 \\ 3 & 7 & 2 \\ 2 & 3 & -7 \end{vmatrix}$

17. $\begin{vmatrix} 2 & -1 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -3 \end{vmatrix}$

18. $\begin{vmatrix} a & -a & a \\ a & a & -a \\ a & -a & -a \end{vmatrix}$

19. $\begin{vmatrix} 1 & 2 & 5 \\ 3 & -4 & 7 \\ 3 & 12 & 15 \end{vmatrix}$

Determinantlarni 3-ustun elementlari bo'yicha yoyib hisoblang:

20. $\begin{vmatrix} 1+\cos a & 1+\sin a & 1 \\ 1-\sin a & 1+\cos a & 1 \\ 1 & 1 & 1 \end{vmatrix}$

21. $\begin{vmatrix} 2\cos^2 a/2 & \sin a & 1 \\ 2\cos^2 b/2 & \sin b & 1 \\ 1 & 0 & 1 \end{vmatrix}$

22. $\begin{vmatrix} \sin a & \cos a & 1 \\ \sin b & \cos b & 1 \\ \sin y & \cos y & 1 \end{vmatrix}$

Qanday shart bajarilganda quyidagi tenglik o'rinli bo'ladi?

23. $\begin{vmatrix} 1 & \cos a & \cos b \\ \cos a & 1 & \cos y \\ \cos b & \cos y & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos a & \cos b \\ \cos a & 0 & \cos y \\ \cos b & \cos y & 0 \end{vmatrix}$

Determinantlarni hisoblang:

24. $\begin{vmatrix} a+x & x & x \\ x & b+x & x \\ x & x & c+x \end{vmatrix}$

25. $\begin{vmatrix} \cos a & \sin a\cos b & \sin a\sin b \\ -\sin a & \cos a\cos b & \cos a\sin b \\ 0 & -\sin b & \cos b \end{vmatrix}$

26. $\begin{vmatrix} 1,(3) & 2,25 \\ 23/3 & 6 \end{vmatrix}$

27. $\begin{vmatrix} \sin 60^0 & \cos 45^0 \\ \sin 45^0 & tg30^0 \end{vmatrix}$

28. $\begin{vmatrix} tga & -1 \\ 4 & ctga \end{vmatrix}$

29. $\begin{vmatrix} (a-1)/2\sqrt{a} & (a+\sqrt{a})/(\sqrt{a}-1) \\ (a\sqrt{a}-\sqrt{a})/2a & (a-\sqrt{a})/(\sqrt{a}+1) \end{vmatrix}$

30. $\begin{vmatrix} 2 & -3 & 1 \\ 6 & -6 & 2 \\ 2 & -1 & 2 \end{vmatrix}$

31. $\begin{vmatrix} 12 & 6 & -4 \\ 6 & 4 & 4 \\ 3 & 2 & 8 \end{vmatrix}$

32. $\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix}$

33. $\begin{vmatrix} m+a & m-a & a \\ n+a & 2n-a & a \\ a & -a & a \end{vmatrix}$

34. $\begin{vmatrix} ax & a^2+x^2 & 1 \\ ay & a^2+y^2 & 1 \\ az & a^2+z^2 & 1 \end{vmatrix}$

35. $\begin{vmatrix} \sin 3a & \cos 3a & 1 \\ \sin 2a & \cos 2a & 1 \\ \sin a & \cos a & 1 \end{vmatrix}$

36. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

37. $\begin{vmatrix} a & x & x \\ x & b & x \\ x & x & c \end{vmatrix}$

38. $\begin{vmatrix} 1 & 5 & 25 \\ 1 & 7 & 49 \\ 1 & 8 & 64 \end{vmatrix}$

39. $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 9 \\ 16 & 25 & 81 \end{vmatrix}$

40. $\begin{vmatrix} 2 & 0 & 3 \\ 7 & 1 & 6 \\ 6 & 0 & 5 \end{vmatrix}$

41. $\begin{vmatrix} \sin x & 0 & -3/2 \\ -2 & 1 & 4 \\ 0,5 & 0 & \cos x \end{vmatrix} = 1$

42. $\begin{vmatrix} 3^x & 2 & -1 \\ 9^x & 2^x & 0 \\ 2^x & 0 & 1 \end{vmatrix} > 0$

Javoblar:

2. -2
3. -35
4. $4\sqrt{ab}$
5. 1
6. $(x^2+y^2)/x(x^2-y^2)$
7. $sin(a+b)sin(a-b)$
8. a) $x_1=1/2,\ x_2=1$

   b) $x_1=-5/2,\quad x_2=3$
9. a) $x \in [2;3]$

   b) $(-\infty ,2)\ \ U\ (5\ ,\ \infty)$
10. 40
11. -10
12. $4a$
13. 68
14. 15
15. 29
16. 0
17. -20
18. $-4a^3$
19. -156
20. 1
21. $sin(b-a)$
22. $sin(b-y)+sin(y-a)+sin(a-b)$
23. $cos^2a+cos^2b+cos^2y=1$
24. $(ab+bc+ca)x+abc$
25. 1
26. -37/4
27. 0
28. 5
29. $-2\sqrt{a},\quad a>0,\quad a\neq 1$
30. 10
31. 72
32. $(x-y)(y-z)(x-z)$
33. amn
34. $a\ (x-z)\ (y-z)(y-x)$
35. $4sinasin^2a/2$
36. $3abc-a^3-b^3-c^3$
37. $2x^3-(a+b+c)x^2+abc$
38. 6
39. 20
40. -8
41. $x=\pi/12+\pi k/2$
42. $x<0$

2. DETERMINANT XOSSALARI .MINOR VA ALGEBRAIK TO’LDIRUVCHILARGA DOIR MISOLLAR

Determinantning asosiy xossalari yordamida yuqori tartibli determinantlar quyi tartibli determinantga kelteriladi.

Misol: a) det = $\begin{vmatrix} 3 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix}$ bu determinantni biror satr yoki ustunda nollar hosil qilib hisoblaymiz. Buning uchun 1-satrni (-1) ga ko’paytirib 2-satrga qo’shamiz:

$$\begin{vmatrix} 3 & 1 & 2 \\ -1 & 0 & -1 \\ 1 & 0 & 2 \end{vmatrix}$$

2 – ustun elementlari bo’yicha yoyib yozamiz:

Det= $1\cdot(-1)^{1+2}\begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix}$ =-(-2+1)=1

b) $\begin{vmatrix} 3 & 5 & 7 & 2 \\ 1 & 2 & 3 & 4 \\ -2 & -3 & 3 & 2 \\ 1 & 3 & 5 & 4 \end{vmatrix}$ Determinantni hisoblang.

Determinantni hisoblash uchun biror yo’l yoki ustunda nollar hosil qilamiz. Buning uchun 2-satr elementlarini (-3) ga ko’paytirib 1-satr elementlariga, 2-satrni 2 ga ko’paytirib 3-satr elementlariga qo’shamiz, 4-satr elementlaridan 2-satr elementlarini ayiramiz. Natijada berilgan determinant quyidagi ko’rinishga keladi:

$$\text{Det} = \begin{vmatrix} 0 & -1 & -2 & -10 \\ 1 & 2 & 3 & 4 \\ 0 & 1 & 9 & 10 \\ 0 & 1 & 2 & 0 \end{vmatrix}$$

Determinantni 1-ustun elementlari bo’yicha yoyib yozamiz:

$$\text{Det} = -\begin{vmatrix} -1 & -2 & -10 \\ 1 & 9 & 10 \\ 1 & 2 & 0 \end{vmatrix}$$

1- satr elementlariga 2- satr elementlarini hadma–had qo'shib, 1 – satr elementlari bo'yicha yoyib yozamiz:

$$\text{Det} = \begin{vmatrix} 0 & 7 & 0 \\ 1 & 9 & 10 \\ 1 & 2 & 0 \end{vmatrix} = 7 \cdot \begin{vmatrix} 1 & 10 \\ 1 & 0 \end{vmatrix} = -70$$

$$\text{Det} = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

n- tartibli determinantning a_{ij} elementining algebraik to'ldiruvchisi

$A_{ij} = (-1)^{i+j} M_{ij}$ formula bo'yicha hisoblanadi, bu yerda M_{ij} a_{ij} elementning minori.

Berilgan $\begin{vmatrix} 3 & -4 & 5 \\ 2 & -3 & 0 \\ 1 & 6 & 10 \end{vmatrix}$ determinantning barcha algebraik to`ldiruvchilarini toping.

$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -3 & 0 \\ 6 & 10 \end{vmatrix} = -30;$ $A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & 0 \\ 1 & 10 \end{vmatrix} = -20;$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & -3 \\ 1 & 6 \end{vmatrix} = 15;$ $A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} -4 & 5 \\ 6 & 10 \end{vmatrix} = 70;$

$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 3 & 5 \\ 1 & 10 \end{vmatrix} = 25;$ $A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 3 & -4 \\ 1 & 6 \end{vmatrix} = -22;$

$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} -4 & 5 \\ -3 & 0 \end{vmatrix} = 15;$ $A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 3 & 5 \\ 2 & 0 \end{vmatrix} = 10;$

$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 3 & -4 \\ 2 & -3 \end{vmatrix} = -1.$

Determinantning ixtiyoriy satr yoki ustun elementlarining o'z algebraik to'ldiruvchilariga ko'paytmalarining yig'indisi uning kattaligiga teng degan xossaga ko'ra, har qanday determinantni ixtiyoriy satr (ustun) bo'yicha yoyib yozish mumkin.

Mustaqil yechish uchun misollar:

1. a) $\begin{vmatrix} 5 & 7 & -1 \\ 2 & 3 & 4 \\ 6 & 1 & 9 \end{vmatrix}$, det, A_{32} ni toping.

b) $\Delta = \begin{vmatrix} 7 & -3 & 0 & 4 \\ 2 & 1 & 1 & 5 \\ 3 & 6 & -1 & -3 \\ 8 & 1 & 1 & 1 \end{vmatrix}$ da A_{41} ni toping.

Determinantlar xossalaridan foydalanib, nollar yig'ib hisoblang:

2. $\begin{vmatrix} 7 & -2 & 3 \\ 0 & 0 & 1 \\ 2 & 1 & -4 \end{vmatrix}$

3. $\begin{vmatrix} 1 & b & 1 \\ 0 & b & 0 \\ b & 0 & -1 \end{vmatrix}$

4. $\begin{vmatrix} -x & 1 & x \\ 0 & -x & -1 \\ x & 1 & -x \end{vmatrix}$

5. $\begin{vmatrix} 5 & 3 & 2 \\ -1 & 2 & 4 \\ 7 & 3 & 6 \end{vmatrix}$

6. $\begin{vmatrix} \sin^2 a & 1 & \cos^2 a \\ \sin^2 b & 1 & \cos^2 b \\ \sin^2 y & 1 & \cos^2 y \end{vmatrix}$

7. $\begin{vmatrix} \sin^2 a & \cos 2a & \cos^2 a \\ \sin^2 b & \cos 2b & \cos^2 b \\ \sin^2 y & \cos 2y & \cos^2 y \end{vmatrix}$

8. $\begin{vmatrix} x & x & ax+bx \\ y & y & ay+by \\ z & z & az+bz \end{vmatrix}$

9. $\begin{vmatrix} a+b & c & 1 \\ b+c & a & 1 \\ c+a & b & 1 \end{vmatrix}$

Determinantlarni qulay usulda hisoblang:

10. $\begin{vmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \\ 2 & 3 & 0 & 1 \end{vmatrix}$

11. $\begin{vmatrix} 1 & 2 & 0 & -3 \\ 3 & 1 & 0 & 4 \\ 1 & 5 & -1 & 7 \\ -2 & 1 & 0 & 1 \end{vmatrix}$

12. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -9 & -9 & -9 & -9 \\ 4 & 3 & 2 & 1 \\ 1 & 0 & 1 & 0 \end{vmatrix}$

13. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 9 \\ 0 & 0 & 3 & 7 \\ -2 & -4 & -6 & 1 \end{vmatrix}$

14. $\begin{vmatrix} 3 & -1 & 2 & -1 & 1 \\ 5 & 1 & -2 & 1 & 2 \\ 9 & -1 & 1 & 3 & 4 \\ 3 & 0 & 6 & -1 & 3 \\ 5 & 2 & 3 & -2 & 1 \end{vmatrix}$

15. $A+B$ ni hisoblang: $A=\begin{vmatrix} 1 & -5 & 2 \\ -2 & 3 & 4 \\ 3 & 2 & 1 \end{vmatrix}$ $B=\begin{vmatrix} 1 & 5 & 2 \\ -2 & -1 & 4 \\ 3 & -2 & 1 \end{vmatrix}$

16. $\begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 6 \\ 0 & 5 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & -2 \\ 0 & 5 & -4 \end{vmatrix}$

17. $A=\begin{vmatrix} 5 & 7 \\ 3 & 4 \end{vmatrix}$ $B=\begin{vmatrix} 1 & 4 \\ 2 & -9 \end{vmatrix}$ $A \cdot B=?$

18. $A=\begin{vmatrix} 7 & 5 \\ 3 & 4 \end{vmatrix}$ $B=\begin{vmatrix} 2 & 9 \\ 1 & 7 \end{vmatrix}$ $A \cdot B=?$

19. $\begin{vmatrix} 7 & 0 & 0 \\ -8 & 1 & -1 \\ 3 & 6 & -4 \end{vmatrix}$

20. $\begin{vmatrix} 4 & -1 & 1 \\ 1 & 2 & 1 \\ -3 & 1 & -2 \end{vmatrix}$

21. $-0{,}125\begin{vmatrix} -1/13 & 2/13 & 0 \\ -3 & 5 & 1 \\ 26 & 26 & 26 \end{vmatrix}$

22. $\begin{vmatrix} 1 & 2 & -3 & 1 \\ 3 & 0 & 1 & -1 \\ 2 & 0 & 4 & 1 \\ 5 & 1 & 2 & 1 \end{vmatrix}$

23. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 9 & 16 \end{vmatrix}$

24. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$

25. $\begin{vmatrix} 0 & 6 & 3 & 5 & 1 \\ -3 & 2 & 4 & 1 & 0 \\ 5 & 1 & 4 & 3 & 2 \\ -3 & 8 & 7 & 6 & 1 \\ 1 & 0 & 3 & 4 & 0 \end{vmatrix}$

26. $\begin{vmatrix} 2 & 3 & 4 \\ 2 & a+3 & b+4 \\ 2 & c+3 & d+4 \end{vmatrix}$

27. $\begin{vmatrix} 1 & -3 & -5 \\ 4 & 2 & 1 \\ 7 & 6 & -6 \end{vmatrix} + \begin{vmatrix} 1 & 3 & -5 \\ 4 & -2 & 1 \\ 7 & 6 & -6 \end{vmatrix}$

28. $\begin{vmatrix} 7 & 8 \\ 5 & 6 \end{vmatrix} * \begin{vmatrix} 9 & 8 \\ 7 & 6 \end{vmatrix}$

29. $\begin{vmatrix} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ 3 & -4 & -1 & -2 \\ 4 & 3 & 2 & -1 \end{vmatrix}$

30. $\begin{vmatrix} -1 & -1 & -1 & -1 \\ -1 & -2 & -4 & -8 \\ -1 & -3 & -9 & -27 \\ -1 & -4 & -16 & -64 \end{vmatrix}$

31. $\begin{vmatrix} 10 & 2 & 0 & 0 & 0 \\ 12 & 10 & 2 & 0 & 0 \\ 0 & 12 & 10 & 2 & 0 \\ 0 & 0 & 12 & 10 & 2 \\ 0 & 0 & 0 & 12 & 10 \end{vmatrix}$

32. $\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1-a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1-b \end{vmatrix}$

Javoblar:

1. a)1 b)-36
2. -11
3. *-b(b+1)*
4. *-2x*
5. 68
6. 0
7. 0
8. 0
9. 0
10. -12
11. 40
12. 0
13. 54
14. 465
15. -10
16. 10
17. 17
18. 65
19. 14
20. -12
21. -1
22. -20
23. 2
24. 160
25. 0
26. *2(ad-bc)*
27. -252
28. -4
29. 900
30. 12
31. 39520
32. a^2b^2

3. MATRITSALAR USTIDA AMALLAR

Matritsalar ustida quyidagi chiziqli amallarni bajarish mumkin.

1. Matritsani songa ko'paytirish uchun uning barcha elementlari shu songa ko'paytiriladi. $k\neq 0$ son hamda

$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ matritsa berilgan bo'lsa, $Ak=\begin{pmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \end{pmatrix}$ tenglik

o'rinli bo`ladi.

3. O'lchamlari bir hil bo'lgan A va B matritsalarni qo'shish uchun mos elementlari qo'shiladi:

$B=\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}$ bo'lsa, $A+B=\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \end{pmatrix}$ matritsa hosil

bo'ladi.

4. Matritsalarni ko'paytirish.

Agar A matritsaning ustunlari soni B matritsaning yo'llar soniga teng bo'lsa A ni B ga ko'paytirish mumkin, nxm o'lchovli $A=(a_{ik})$ matritsani mxp o'lchovli $B=(b_{ik})$ matritsaga quyidagi formula bo'yicha ko'paytiriladi.

$$c_{ik}=\sum_{j=1}^{n} a_{ij}b_{ik}$$

Amallarni bajaring:

1. $A=\begin{bmatrix} 2 & 4 & 1 \\ -1 & 0 & 2 \end{bmatrix}$ $B=\begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ $A+B$ matritsani toping.

$$A+B=\begin{bmatrix} 2 & 4 & 1 \\ -1 & 0 & 2 \end{bmatrix}+\begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}=\begin{bmatrix} 2+0 & 4+2 & 1+1 \\ -1+1 & 0+1 & 2+2 \end{bmatrix}=\begin{bmatrix} 2 & 6 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

2. $A=\begin{bmatrix} 7 & -12 \\ -4 & 7 \end{bmatrix}$ $B=\begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$ $A\cdot B$ matritsani toping.

$$A\cdot B=\begin{bmatrix} 7 & -12 \\ -4 & 7 \end{bmatrix}\cdot\begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}=\begin{bmatrix} 7.26+(-12).15 & 7.45+(-12).26 \\ -4.26+7.15 & -4.45+7.26 \end{bmatrix}=\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Mustaqil yechish uchun misollar:

Berilgan matritsalar ustida talab qilingan amallani bajaring.

3. $A=\begin{bmatrix} 1 & 5 \\ 2 & -4 \end{bmatrix}$ $B=\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$ $2A-B=?$

4. $A=\begin{bmatrix} 1 & -1 & -3 \\ 2 & 1 & 5 \end{bmatrix}$ $B=\begin{bmatrix} 0 & 3 & 2 \\ -1 & 4 & 1 \end{bmatrix}$ $3A-2B=?$

5. $\begin{bmatrix} 7 & 0 \\ 3 & 1 \\ -1 & 2 \end{bmatrix}-3\begin{bmatrix} 2 & \sqrt{2} \\ 1 & -1 \\ -1 & 0 \end{bmatrix}+\begin{bmatrix} 1 & \sqrt{18} \\ 4 & -5 \\ 3 & 1 \end{bmatrix}$

6. $C=(1\ \ 2\ \ 3)$, $F=\begin{bmatrix} 4 & -3 \\ 1 & 2 \\ 0 & 2 \end{bmatrix}$ $C*F=?$

7. $A=\begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, $B=\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ $A*B=?$

8. $A=\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 4 \\ -4 & 5 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 4 & 1 \\ 0 & 2 & 5 \\ 1 & -1 & 4 \end{bmatrix}$ $A*B=?$

9. $A=\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$, $A^2=?$

10. $A=\begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \\ 4 & 1 & 1 \end{bmatrix}$, E-birlik matritsa $2A^2+3A+5E=?$

11. $A=\begin{bmatrix} 3 & 4 & 2 \\ 1 & 0 & 5 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 0 \\ 1 & 3 \\ 0 & 5 \end{bmatrix}$, $C=\begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$ $A*B-C^2=?$

12. $A=\begin{bmatrix} 1 & 2 & -3 \\ 1 & 0 & 2 \\ 4 & 5 & 3 \end{bmatrix}$, $B=\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $C=(2\ \ 0\ \ 5)$, E- birlik matritsa $A*B*C-3E=?$

13. $A=\begin{pmatrix} 2 & 0 & 1 \\ -2 & 3 & 2 \\ 4 & -1 & 5 \end{pmatrix}$, $B=\begin{pmatrix} -3 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 3 \end{pmatrix}$ $A*B=?$

14. $\begin{pmatrix} 1 & -3 & 2 \\ 3 & -4 & 1 \\ 2 & -5 & 3 \end{pmatrix} * \begin{pmatrix} 2 & 5 & 6 \\ 1 & 2 & 5 \\ 1 & 3 & 2 \end{pmatrix} =?$

15. $\begin{pmatrix} 2 & -1 & 3 & -4 \\ 3 & -2 & 4 & -3 \\ 5 & -3 & -2 & 1 \\ 3 & -3 & 1 & 2 \end{pmatrix} * \begin{pmatrix} 7 & 8 & 6 & 9 \\ 5 & 7 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 2 & 1 & 1 & 2 \end{pmatrix} =?$

16. $\begin{pmatrix} 5 & 7 & -3 & -4 \\ 7 & 6 & -4 & -5 \\ 6 & 4 & -3 & -2 \\ 8 & 5 & -6 & -1 \end{pmatrix} * \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{pmatrix} =?$

17. $A=\begin{pmatrix} 3 & 5 \\ 4 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$ $2A+5B=?$

18. $A=\begin{pmatrix} 3 & 5 & 7 \\ 2 & -1 & 0 \\ 4 & 3 & 2 \end{pmatrix}$ $B=\begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & -2 \\ -1 & 0 & 1 \end{pmatrix}$ $A+B=?$

19. $A=\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 5 \end{pmatrix}$ $C=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $A*C=?$

20. $A=\begin{pmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}$ $F=\begin{pmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 0 \end{pmatrix}$ $A*F=?$

21. $A=\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$ $B=\begin{pmatrix} 5 & 7 \\ -1 & 2 \end{pmatrix}$ $A^2-A*B+2BA=?$

22. $A=\begin{pmatrix} 1 & -3 & 0 \\ 2 & 5 & 1 \end{pmatrix}$ $B=\begin{pmatrix} 0 & -1 & 3 \\ 3 & 5 & 2 \\ 4 & -2 & 1 \end{pmatrix}$ $A*B=?$

23. $A=\begin{pmatrix} 1 & 3 & 1 \\ 2 & 0 & 4 \\ 1 & 2 & 3 \end{pmatrix}$ $B=\begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$ $A*B=?$ $B*A=?$

24. $A=\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ $\quad A^2+A+E=?$

25. $A=\begin{pmatrix} 4 & 3 \\ 7 & 5 \end{pmatrix}$ $\quad B=\begin{pmatrix} -28 & 93 \\ 38 & -126 \end{pmatrix}$ $\quad C=\begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}$ $\quad A*B*C=?$

26. $\begin{pmatrix} 1 & 3 \\ 2 & 0 \\ 1 & -1 \end{pmatrix} * \begin{pmatrix} 1 & -2 & 3 \\ 5 & 4 & 0 \end{pmatrix} + \begin{pmatrix} -10 & -9 & 7 \\ 1 & 5 & 8 \\ -1 & -3 & 6 \end{pmatrix} = ?$

27. $\begin{pmatrix} 5 & 8 & -4 \\ 6 & 9 & -5 \\ 4 & 7 & -3 \end{pmatrix} * \begin{pmatrix} 3 & 2 & 5 \\ 4 & -1 & 3 \\ 9 & 6 & 5 \end{pmatrix} = ?$

28. $\begin{pmatrix} 4 & 3 \\ 7 & 5 \end{pmatrix} * \begin{pmatrix} -28 & 93 \\ 38 & -126 \end{pmatrix} * \begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix} = ?$

29. $\begin{pmatrix} 5 & 2 & -2 & 3 \\ 6 & 4 & -3 & 5 \\ 9 & 2 & -3 & 4 \\ 7 & 6 & -4 & 7 \end{pmatrix} * \begin{pmatrix} 2 & 2 & 2 & 2 \\ -1 & -5 & 3 & 11 \\ 16 & 24 & 8 & -8 \\ 8 & 16 & 0 & -16 \end{pmatrix} = ?$

30. $\begin{pmatrix} 1 & 1 & 1 & -1 \\ -5 & -3 & -4 & 4 \\ 5 & 1 & 4 & -3 \\ -16 & -11 & -15 & 14 \end{pmatrix} * \begin{pmatrix} 7 & -2 & 3 & 4 \\ 11 & 0 & 3 & 4 \\ 5 & 4 & 3 & 0 \\ 22 & 2 & 9 & 8 \end{pmatrix} = ?$

Javoblar:

3. $\begin{pmatrix} -1 & 8 \\ 0 & -9 \end{pmatrix}$

4. $\begin{pmatrix} 3 & -9 & -13 \\ 8 & -5 & 13 \end{pmatrix}$

5. $\begin{pmatrix} 2 & 0 \\ 4 & -1 \\ 5 & 3 \end{pmatrix}$

6. $(6 \quad 7)$

7. $\begin{pmatrix} 1 & -1 \\ 0 & 1 \\ -1 & 0 \end{pmatrix}$

8. $\begin{pmatrix} 5 & 0 & 4 \\ 10 & 10 & 33 \\ -11 & -7 & 25 \end{pmatrix}$

9. $\begin{pmatrix} 11 & 14 \\ 7 & 18 \end{pmatrix}$

10. $\begin{pmatrix} 28 & 15 & 16 \\ 19 & 36 & 15 \\ 30 & 19 & 28 \end{pmatrix}$

11. $\begin{pmatrix} 9 & 7 \\ 2 & 9 \end{pmatrix}$

12. $\begin{pmatrix} 1 & 0 & 10 \\ 6 & -3 & 15 \\ 34 & 0 & 82 \end{pmatrix}$

13. $\begin{pmatrix} -6 & 1 & 3 \\ 6 & 2 & 9 \\ -12 & -3 & 14 \end{pmatrix}$

14. $\begin{pmatrix} 1 & 5 & -5 \\ 3 & 10 & 0 \\ 2 & 9 & -7 \end{pmatrix}$

15. $\begin{pmatrix} 10 & 17 & 19 & 23 \\ 17 & 23 & 27 & 35 \\ 16 & 12 & 9 & 20 \\ 7 & 1 & 3 & 10 \end{pmatrix}$

16. $\begin{pmatrix} 8 & 6 & 4 & 2 \\ 5 & 0 & -5 & -10 \\ 7 & 7 & 7 & 7 \\ 10 & 9 & 8 & 7 \end{pmatrix}$

17. $\begin{pmatrix} 16 & 25 \\ 13 & -8 \end{pmatrix}$

18. $\begin{pmatrix} 4 & 7 & 11 \\ 4 & 2 & -2 \\ 3 & 3 & 3 \end{pmatrix}$

19. $\begin{pmatrix} 8 \\ 19 \end{pmatrix}$

20. $\begin{pmatrix} 6 & 10 \\ 6 & 5 \\ 2 & 3 \end{pmatrix}$

21. $\begin{pmatrix} 73 & 25 \\ 1 & -11 \end{pmatrix}$

22. $\begin{pmatrix} -9 & -16 & -3 \\ 19 & 21 & 17 \end{pmatrix}$

23. $\begin{pmatrix} 8 & 0 & 7 \\ 16 & 10 & 4 \\ 13 & 5 & 7 \end{pmatrix}$; $\begin{pmatrix} 4 & 6 & 6 \\ 1 & 7 & 3 \\ 8 & 11 & 14 \end{pmatrix}$

24. $\begin{pmatrix} 9 & 6 & 6 \\ 6 & 9 & 6 \\ 6 & 6 & 9 \end{pmatrix}$

25. $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

26. $\begin{pmatrix} 6 & 1 & 10 \\ 3 & 1 & 14 \\ -5 & -9 & 9 \end{pmatrix}$

27. $\begin{pmatrix} 11 & -22 & 29 \\ 9 & -27 & 32 \\ 13 & -17 & 26 \end{pmatrix}$

28. $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

29. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

30. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$

4. MATRITSA RANGINI HISOBLASH. TESKARI MATRITSANI TOPISH

1. $A=\begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{pmatrix}$ (1)

A matritsaning rangi deb noldan farqli minorlarning eng yuqori tartibiga aytiladi va *rang(A)* kabi ifodalanadi.

Matritsa rangi ikki usulda topiladi:

1. Matritsa rangi ta'rifga asoslangan "minorlar ajratish" usuli;
2. Matritsa ustun va satrlarida nollar yig'ib hisoblashga asoslangan "Gauss algoritmi".

Misol 1. Matritsa rangini hisoblang:

$A=\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}$ *A* matritsa 3x4 tartibli, demak uning rangi 3 dan yuqori bo'lmaydi. Uchinchi tartibli minorlarni hisoblaymiz:

$M_1=\begin{vmatrix} 2 & -1 & 3 \\ 4 & -2 & 5 \\ 2 & -1 & 1 \end{vmatrix}$=-4-10-12+12+4+10=0 $M_2=\begin{vmatrix} 2 & -1 & -2 \\ 4 & -2 & 1 \\ 2 & -1 & 8 \end{vmatrix}$=-32-2+8-8+32+2=0

$M_3=\begin{vmatrix} 2 & -1 & 4 \\ 4 & -2 & 7 \\ 2 & -1 & 2 \end{vmatrix}$=-8-14-16+16+8-14=0 $M_4=\begin{vmatrix} -1 & 3 & -2 \\ -2 & 5 & 1 \\ -1 & 1 & 8 \end{vmatrix}$=-40-3+4-10+48+1=0

$M_5=\begin{vmatrix} 3 & -2 & 4 \\ 5 & 1 & 7 \\ 1 & 8 & 2 \end{vmatrix}$=6-14+160-4+20-168=0

Barcha uchinchi tartibli minorlar nolga teng. Ikkinchi tartibli minorlarni hisoblaymiz:

$M_1^1=\begin{vmatrix} -1 & 3 \\ -2 & 5 \end{vmatrix}$=-5+6=1 $M_1^1 \neq 0$ *r(A)*=2

Misol 2. Matritsa rangini elementar almashtirishlar yordamida nollar yig'ib hisoblaymiz:

$$A=\begin{pmatrix}25 & 31 & 17 & 43\\ 75 & 94 & 53 & 132\\ 75 & 94 & 54 & 134\\ 25 & 32 & 20 & 48\end{pmatrix} \sim \begin{pmatrix}25 & 31 & 17 & 43\\ 0 & 1 & 2 & 3\\ 0 & 1 & 3 & 5\\ 0 & 1 & 3 & 5\end{pmatrix} \sim \begin{pmatrix}25 & 31 & 17 & 43\\ 0 & 1 & 2 & 3\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0\end{pmatrix}$$

bu matritsaning rangi $\begin{pmatrix}31 & 17 & 43\\ 1 & 2 & 3\\ 0 & 1 & 2\end{pmatrix}$ matritsa rangiga teng.

$$\begin{vmatrix}31 & 17 & 43\\ 1 & 2 & 3\\ 0 & 1 & 2\end{vmatrix}=40\neq 0 \qquad r\begin{pmatrix}31 & 17 & 43\\ 1 & 2 & 3\\ 0 & 1 & 2\end{pmatrix}=3.$$

Demak, berilgan matritsaning rangi ham 3 ga teng. $r(A)=3$

(1) ko`rinishdagi A matritsa uchun teskari matritsa 2 usulda topiladi:

1. Klassik usuli;
2. Jordan usuli.

Misol 3. $A=\begin{pmatrix}2 & 3 & 2\\ 5 & 1 & 4\\ 1 & -2 & -1\end{pmatrix}$ matritsa uchun teskari A^{-1} matritsani klassik usulda toping.

Klassik usulda teskari matritsa $A^{-1}=1/|A|\begin{pmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{pmatrix}$ (2)

formula bo'yicha hisoblanadi. Bu yerda $|A|$ berilgan matritsa determinanti. A_{ij}(i=1, 2, 3; j=1, 2, 3) transponirlangan matritsaning algebraik to'ldiruvchilari.

$|A|=\begin{vmatrix}2 & 3 & 2\\ 5 & 1 & 4\\ 1 & -2 & -1\end{vmatrix}$=-2+12-20-2+15+16=43-24=19≠0. Demak, A matritsa maxsusmas matritsa. A^{-1} teskari matritsa mavjud. Algebraik to'ldiruvchilarini hisoblaymiz:

A_{11}=$\begin{vmatrix} 1 & 4 \\ -2 & -1 \end{vmatrix}$=-1+8=7 A_{21}=-$\begin{vmatrix} 3 & 2 \\ -2 & -1 \end{vmatrix}$=-(-3+4)=-1

A_{31}=$\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$=12-2=10 A_{12}=-$\begin{vmatrix} 5 & 4 \\ 1 & -1 \end{vmatrix}$=-(-5-4)=9

A_{22}=$\begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix}$=-2-2=-4 A_{32}=-$\begin{vmatrix} 2 & 2 \\ 5 & 4 \end{vmatrix}$=-(8-10)=2

A_{13}=$\begin{vmatrix} 5 & 1 \\ 1 & -2 \end{vmatrix}$=-10-1=-11 A_{23}=-$\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix}$=-(-4-3)=7

A_{33}=$\begin{vmatrix} 2 & 3 \\ 5 & 1 \end{vmatrix}$=2-15=-13

A_{ij} larni (2) formulaga qo'yamiz:

A^{-1}=1/19$\begin{pmatrix} 7 & -1 & 10 \\ 9 & -4 & 2 \\ -11 & 7 & -13 \end{pmatrix}$ teskari matritsaning to'g'ri topilganini

$AA^{-1}=E$ (3)

formula bo'yicha tekshiramiz:

$$\begin{pmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 1 & -2 & -1 \end{pmatrix} * 1/19 \begin{pmatrix} 7 & -1 & 10 \\ 9 & -4 & 2 \\ -11 & 7 & -13 \end{pmatrix} = 1/19 * \begin{pmatrix} 14+27-22 & -2-12+14 & 20+6-26 \\ 35+9-44 & -5-4+28 & 50+2-52 \\ 7-18+11 & -1+8-7 & 10-4+13 \end{pmatrix} =$$

$$=1/19 * \begin{pmatrix} 19 & 0 & 0 \\ 0 & 19 & 0 \\ 0 & 0 & 19 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E$$

Misol 4. $A=\begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & -3 \\ 4 & 3 & -2 \end{pmatrix}$

$|A|=16\neq0$ teskari matritsa mavjud. Teskari matritsani Jordan usulida topamiz. Berilgan matritsani birlik matritsa hisobida kengaytirib, elementar almashtirishlar bajaramiz, bu usulni to chap tomonda A matritsa o'rnida birlik matritsa hosil bo'lguncha davom ettiramiz, o'ng tomonda hosil bo'lgan matritsa berilgan matritsaga nisbatan teskari matritsa bo'ladi.

$\left(A|E \sim E|A^{-1}\right)$- Jordan usuli algaritmi.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 1 \\ -1 & -1 & -3 & 0 & 1 & 0 \\ 4 & 3 & -2 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ 0 & -5 & -6 & -4 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ 0 & 0 & -16 & 1 & 5 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 5 & -1 & -2 & 0 \\ 0 & 1 & 0 & 14/16 & 6/16 & -2/16 \\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -11/16 & -7/16 & 5/16 \\ 0 & 1 & 0 & 14/16 & 6/16 & -2/16 \\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16 \end{array}\right) \qquad A^{-1}=1/16\begin{pmatrix} -11 & -7 & 5 \\ 14 & 6 & -2 \\ -1 & -5 & -1 \end{pmatrix}$$

teskari matritsa to'g'ri topilganini (3) formulaga qo'yib tekshiramiz:

$$AA^{-1}=1/16\begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & -3 \\ 4 & 3 & -2 \end{pmatrix} * \begin{pmatrix} -11 & -7 & -5 \\ 14 & 6 & -2 \\ -1 & -5 & -1 \end{pmatrix} =$$

$$=1/16\begin{pmatrix} -11+28-1 & -7+12-5 & 5-4-1 \\ 11-14+3 & 7-6+15 & -5+2+3 \\ -44+42+2 & -28+18+10 & 20-6+2 \end{pmatrix} =$$

$$=1/16\begin{pmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ demak, teskari matritsa to'gri topilgan.

Mustaqil yechish uchun misollar:

Berilgan kvadrat matritsaning determinantlari, normalari va ranglari topilsin:

1. a) $A=\begin{pmatrix} 1 & 2 \\ -2 & 0 \end{pmatrix}$ b) $A=\begin{pmatrix} -1 & 0 & 8 \\ 5 & 9 & 0 \\ 0 & 4 & 3 \end{pmatrix}$

c) $A=\begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 2 \end{pmatrix}$ d) $A=\begin{pmatrix} 2 & 3 & 4 & 0 \\ 1 & 5 & 7 & 0 \\ 3 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Quyidagi matritsalar rangini minorlar ajratish usuli bilan hisoblang:

2. $A=\begin{pmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{pmatrix}$ 3. $A=\begin{pmatrix} 2 & 1 & -2 & 3 \\ -2 & 9 & -4 & 7 \\ -4 & 3 & 1 & -1 \end{pmatrix}$

4. $A=\begin{pmatrix} 3 & 5 & 7 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}$ 5. $A=\begin{pmatrix} 0 & 2 & -4 \\ -1 & -4 & 5 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \\ 2 & 3 & 0 \end{pmatrix}$

6. $A=\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}$ 8. $A=\begin{pmatrix} 1 & 2 & 1 & 3 \\ 4 & -1 & -5 & -6 \\ 1 & -3 & -4 & -7 \\ 2 & 1 & -1 & 0 \end{pmatrix}$

Quyidagi matritsalar rangini elementar almashtirish usuli bilan hisoblang:

9. $\begin{pmatrix} 1 & 2 & 1 & 3 & 4 \\ 3 & 4 & 2 & 6 & 8 \\ 1 & 2 & 1 & 8 & 4 \end{pmatrix}$ 10. $\begin{pmatrix} 1 & 7 & 5 & 8 & 9 & 2 \\ 3 & 21 & 15 & 24 & 27 & 6 \\ 2 & 14 & 10 & 16 & 18 & 4 \end{pmatrix}$

11. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix}$ 12. $\begin{pmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 2 & 0 & 4 & 0 & 0 \end{pmatrix}$

13. $\begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 8 & 6 & -7 & 4 & 2 \\ 4 & 3 & -8 & 2 & 7 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & -6 \end{pmatrix}$ 14. $\begin{pmatrix} 24 & 19 & 36 & 72 & -38 \\ 49 & 40 & 73 & 147 & -80 \\ 73 & 59 & 98 & 219 & -118 \\ 47 & 36 & 71 & 141 & -72 \end{pmatrix}$

15. $\begin{pmatrix} 17 & -28 & 45 & 11 & 39 \\ 24 & -37 & 61 & 13 & 50 \\ 25 & -7 & 32 & -18 & -11 \\ 31 & 12 & 19 & -43 & -55 \\ 42 & 13 & 29 & -55 & -68 \end{pmatrix}$ 16. $\begin{pmatrix} 47 & -67 & 35 & 201 & 155 \\ 26 & 98 & 23 & -294 & 6 \\ 16 & -428 & 1 & 1284 & 52 \end{pmatrix}$

Berilgan kvadrat matritsalar uchun teskari matritsani ikki usulda toping:

17. $\begin{pmatrix} -1 & 1 \\ 4 & 2 \end{pmatrix}$

18. $\begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$

19. $\begin{pmatrix} tga & 1 \\ 2 & ctga \end{pmatrix}$

20. $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 2 \end{pmatrix}$

21. $\begin{pmatrix} 1 & -1 & 1 \\ -38 & 41 & -34 \\ 27 & -29 & 24 \end{pmatrix}$

22. $\begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}$

23. $\begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix}$

Quyidagi matritsali tenglamalarni eching:

24. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} X = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix}$

25. $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} X = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}$

Berilgan matritsalarning determinantlari va normalari topilsin:

26. a) $A = \begin{pmatrix} 2 & 5 \\ -4 & 2 \end{pmatrix}$

b) $A = \begin{pmatrix} 1 & 0 & 5 \\ 4 & -2 & -1 \\ 2 & 1 & 3 \end{pmatrix}$

c) $A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 2 \end{pmatrix}$

d) $A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 1 & 1 & 3 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 0 \end{pmatrix}$

Matritsalarning ranglari topilsin:

27. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix}$

28. $\begin{pmatrix} 1 & 2 & 3 & 6 \\ 2 & 3 & 1 & 6 \\ 3 & 1 & 2 & 6 \end{pmatrix}$

29. $\begin{pmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & 3 & 0 \end{pmatrix}$

30. $\begin{pmatrix} 4 & 5 & 2 & 1 & -3 \\ 0 & 2 & 1 & 1 & 2 \\ 4 & 7 & 3 & 3 & -1 \\ 8 & 12 & 5 & 3 & -4 \end{pmatrix}$

31. $\begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{pmatrix}$

32. $\begin{pmatrix} 3 & -1 & 3 & 2 & 5 \\ 5 & -3 & 2 & 3 & 4 \\ 1 & -3 & -5 & 0 & -7 \\ 7 & -5 & 1 & 4 & 1 \end{pmatrix}$

Matritsaning teskarisini toping:

33. $\begin{pmatrix} 1 & 5 & 7 \\ 3 & 1 & 1 \\ 2 & 3 & 4 \end{pmatrix}$

34. $\begin{pmatrix} 2 & -1 & 7 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{pmatrix}$

35. $\begin{pmatrix} 1 & 0 & -2 \\ 3 & 1 & 0 \\ -1 & 2 & 4 \end{pmatrix}$

36. $\begin{pmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{pmatrix}$

Quyidagi matritsali tenglamani eching:

37. $\begin{pmatrix} 1 & -3 \\ 4 & -6 \end{pmatrix} X = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$

Javoblar:

1. a) $|A|=4$, $N(A)=3$, $r(A)=2$ b) $|A|=133$, $N(A)=14$, $r(A)=3$

c) $|A|=0$, $N(A)=\sqrt{29}$, $r(A)=2$ d) $|A|=0$, $N(A)=\sqrt{116}$, $r(A)=2$

2. $r=3$ 3. $r=2$

4. $r=2$ 5. $r=2$

6. $r=2$ 7. $r=2$

8. $r=2$ 9. $r=1$

10. $r=1$ 11. $r=2$

12. $r=2$ 13. $r=3$

14. $r=2$ 15. $r=2$

16. $\begin{pmatrix} 1 & 0,5 \\ 2 & 0,5 \end{pmatrix}$ 17. A^{-1} mavjud emas.

18. $\begin{pmatrix} -ctga & 1 \\ 2 & -tga \end{pmatrix}$

19. $\begin{pmatrix} -2 & -1 & 2 \\ 4 & 1 & -3 \\ 1 & 1 & -1 \end{pmatrix}$

20. $\begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}$

21. $\begin{pmatrix} 1 & -1 & 1 \\ -38 & 41 & -34 \\ 27 & -29 & 24 \end{pmatrix}$

22. $\begin{pmatrix} -8 & 29 & -11 \\ -5 & 18 & -7 \\ 1 & -3 & 1 \end{pmatrix}$

23. $\begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix}$

24. $\begin{pmatrix} 6 & 4 & 5 \\ 2 & 1 & 2 \\ 3 & 3 & 3 \end{pmatrix}$

25. a) $|A|=24$, $N(A)=7$, $r(A)=2$ b) $|A|=35$, $N(A)=\sqrt{61}$, $r(A)=2$

c) $|A|=0$, $N(A)=\sqrt{29}$, $r(A)=2$ d) $|A|=0$, $N(A)=6$, $r(A)=3$

26. $r(A)=2$

27. $r(A)=3$

28. $r(A)=3$

29. $r(A)=2$

30. $r=3$

31. $r=3$

32. Teskari matritsa mavjud emas.

33. $C^{-1}=\begin{pmatrix} 1/134 & 31/134 & -23/134 \\ 13/134 & -1/134 & 31/134 \\ 17/134 & -9/134 & 11/134 \end{pmatrix}$

34. $\begin{pmatrix} -5/2 & -5/2 & -1/5 \\ -6/5 & -4/5 & -4/5 \\ -7/10 & -1/5 & -1/10 \end{pmatrix}$

35. $\begin{pmatrix} 9/5 & -2/5 & -4/5 \\ 1/5 & 2/5 & -1/5 \\ -12/5 & 1/5 & 7/5 \end{pmatrix}$

36. $\begin{pmatrix} 0 & 0 \\ 3/2 & 1 \end{pmatrix}$

5. CHIZIQLI TENGLAMALAR SISTEMASINING YECHIMI HAQIDA KRONEKER - KAPELLI TEOREMASI

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1m}x_m = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2m}x_m = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nm}x_m = b_n \end{cases} \qquad (1)$$

Kroneker-Kapelli teoremasi (1) chiziqli tenglamalar sistemasining birgalikda yoki birgalikda emasligini aniqlaydi.

(1) chiziqli tenglamalar sistemasining asosiy va ozod hadlar hisobiga kengaytirilgan matritsasini tuzamiz:

$$A=\begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1m} \\ a_{21} & a_{22} & \ldots & a_{2m} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nm} \end{pmatrix} \qquad (2)$$

$$B=\begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1m} & |b_1 \\ a_{21} & a_{22} & \ldots & a_{2m} & |b_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nm} & |b_n \end{pmatrix} \qquad (3)$$

Teorema. Agar *A* matritsa rangi *B* matritsa rangiga teng bo'lib, noma'lumlar soniga ham teng bo'lsa, ya'ni *r(A)=r(B)=m* bo'lsa, (1) tenglamalar sistemasi aniq bo'ladi, sistema birgalikda bo'lib yagona yechimga ega bo'ladi.

Agar *r(A)=r(B)<m* bo'lsa, (1) sistema birgalikda bo'lib, cheksiz ko'p yechimga ega bo'ladi.

Agar *r(A)<r(B)* bo'lsa, sistema birgalikda bo'lmaydi, sistema yechimga ega bo'lmaydi.

Misollar ko'ramiz: 1. Quyidagi sistemalarni birgalikda yoki birgalikda emasligini tekshiramiz:

a) $\begin{cases} x_1 + 3x_2 + 5x_3 + 7x_4 + 9x_5 = 1 \\ x_1 - 2x_2 + 3x_3 - 4x_4 + 5x_5 = 2 \\ 2x_1 + 11x_2 + 12x_3 + 25x_4 + 25x_5 = 4 \end{cases}$

Buning uchun asosiy va kengaytirilgan matritsa rangini topamiz:

$$A=\begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 1 & -2 & 3 & -4 & 5 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 3 & 9 & 15 & 21 & 27 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 1 & 3 & 5 & 7 & 9 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix}$$

2- satr elementlaridan 1- satr elementlarini ayiramiz:

$$A \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix} \qquad r(A)=2$$

$$B=\begin{pmatrix} 1 & 3 & 5 & 7 & 9 & |1 \\ 1 & -2 & 3 & -4 & 5 & |2 \\ 2 & 11 & 12 & 25 & 22 & |4 \end{pmatrix}$$

bu matritsa rangini topish uchun yana yuqoridagi ishni takrorlaymiz, natijada B matritsa quyidagi ko'rinishni oladi.

$$B \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 & |1 \\ 0 & 0 & 0 & 0 & 0 & |1 \\ 2 & 11 & 12 & 25 & 22 & |4 \end{pmatrix}, \qquad B_1 = \begin{pmatrix} 7 & 9 & 1 \\ 0 & 0 & 1 \\ 25 & 22 & 4 \end{pmatrix}$$

matritsa rangini topamiz:

$$M = |B_1| = \begin{vmatrix} 7 & 9 & 1 \\ 0 & 0 & 1 \\ 25 & 22 & 4 \end{vmatrix} = 225 - 154 = 71; \qquad r(B_1) = 3$$

Demak, $r(B)=3$ bo'lib, $r(A)\neq r(B)$ sistema birgalikda emas.

b) $\begin{cases} x_1 + 5x_2 + 4x_3 + 3x_4 = 1 \\ 2x_1 - x_2 + 2x_3 - x_4 = 0 \\ 5x_1 + 3x_2 + 8x_3 + x_4 = 1 \end{cases}$

Sistema birgalikda yoki birgalikda emasligini tekshiring.

Ozod hadlar hisobiga kengaytirilgan matritsa tuzamiz:

$$B=\left(\begin{array}{cccc|c}1&5&4&3&1\\2&-1&2&-1&0\\5&3&8&1&1\end{array}\right)$$

3- satr elementlaridan 1- satr elementlarini ayiramiz:

$$B=\left(\begin{array}{cccc|c}1&5&4&3&1\\2&-1&2&-1&0\\4&-2&4&-2&0\end{array}\right)\sim\left(\begin{array}{cccc|c}1&5&4&3&1\\2&-1&2&-1&0\\2&-1&2&-1&0\end{array}\right)\sim\left(\begin{array}{cccc|c}1&5&4&3&1\\2&-1&2&-1&0\\0&0&0&0&0\end{array}\right)$$

r(A)=r(B)=2 ekanini ko'rish mumkin. Demak, sistema birgalikda.

Mustaqil yechish uchun misollar:

Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring:

2. $\begin{cases}x_1+x_2+x_3=1\\x_1+x_2+2x_3=1\\x_1+x_2+3x_3=2\end{cases}$

3. $\begin{cases}x_1+x_2+x_3=1\\x_1+x_2+2x_3=1\\2x_1+2x_2+4x_3=1\end{cases}$

4. $\begin{cases}2x+3y+2z=9\\x+2y-3z=14\\3x+4y+z=16\end{cases}$

5. $\begin{cases}x_1+2x_2-x_3=1\\x_1+4x_2-3x_3=7\end{cases}$

6. $\begin{cases}2x_1-x_2=3\\3x_1-5x_2=1\\4x_1-7x_2=1\end{cases}$

7. $\begin{cases}x_1+3x_2+5x_3-2x_4=3\\x_1+4x_2-2x_3+3x_4=2\\-x_1-2x_2-12x_3-7x_4=-4\\3x_1+11x_2+x_3-4x_4=7\end{cases}$

8. $\begin{cases}3x_1+2x_2=4\\x_1-4x_2=-1\\7x_1+10x_2=12\\5x_1+6x_2=8\\3x_1-16x_2=-5\end{cases}$

9. $\begin{cases}x_1+5x_2+4x_3=1\\2x_1+10x_2+8x_3=3\\3x_1+15x_2+12x_3=5\end{cases}$

10. $\begin{cases}x_1-3x_2+2x_3=-1\\x_1+9x_2+6x_3=3\\x_1+3x_2+4x_3=1\end{cases}$

11. $\begin{cases}x_1+2x_2-4x_3=18\\3x_1-x_2+4x_3=4\\2x_1+x_2+5x_3=0\end{cases}$

12. $\begin{cases}2x_1+x_2+3x_3=5\\x_1-3x_2+5x_3=4\\2x_1+x_2+5x_3=0\end{cases}$

13. $\begin{cases}x_1+2x_2-x_3=1\\2x_1+4x_2-3x_3=2\end{cases}$

14. $\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 2x_1 + 4x_2 - 3x_3 = 5 \end{cases}$

15. $\begin{cases} x_1 - 3x_2 + 5x_3 + 7x_4 + 9x_5 = 1 \\ x_1 - x_2 + 3x_3 - 4x_4 + 5x_5 = 2 \\ 2x_1 + 11x_2 + 12x_3 + 25x_4 + 22x_5 = 4 \end{cases}$

16. $\begin{cases} x_1 + 2x_2 + 3x_3 = 14 \\ 3x_1 + 2x_2 + x_3 = 10 \\ x_1 + x_2 + x_3 = 6 \\ 2x_1 + 3x_2 - x_3 = 5 \\ x_1 + x_2 = 3 \end{cases}$

17. $\begin{cases} 3x_1 + 2x_2 = 4 \\ x_1 - 4x_2 = -1 \\ 7x_1 + 10x_2 = 12 \\ 5x_1 + 6x_2 = 8 \\ 3x_1 - 16x_2 = 5 \end{cases}$

18. $\begin{cases} 3x_1 - 5x_2 + 2x_3 + 4x_4 = 2 \\ 7x_1 - 4x_2 + x_3 + 3x_4 = 5 \\ 5x_1 + 7x_2 - 4x_3 - 6x_4 = 3 \end{cases}$

Javoblar:

2. *r(A)=2, r(B)=3* sistema birgalikda emas.

3. *r(A)=3, r(B)=3* sistema birgalikda emas.

4. *r(A)=r(B)* sistema birgalikda.

5. *r(A)=r(B)* sistema birgalikda.

6. Sistema birgalikda.

7. *r(A)=r(B)=3* sistema birgalikda.

8. *r(A)=r(B)=2* sistema birgalikda.

9. *r(A)=2, r(B)=3* sistema birgalikda emas.

10. *r(A)=r(B)=2* sistema birgalikda.

11. Sistema birgalikda.

12. Sisema birgalikda.

13. *r(A)=r(B)=2* sistema birgalikda.

14. *r(A)=2, r(B)=2* sistema birgalikda.

15. *r(A)=2, r(B)=3* sistema birgalikda emas.

16. *r(A)=r(B)=3* sistema birgalikda.

17. *r(A)=r(B)=2* sistema birgalikda.

18. Sistema birgalikda emas.

6. CHIZIQLI TENGLAMALAR SISTEMASINI KRAMER HAMDA TESKARI MATRITSA USULI BILAN YECHISH

1. Chiziqli tenglamalar sistemasini yechishning Kramer formulasi determinantlardan foydalanib sistema yechimini topishdir.

Sistema yechimi Kramer formulalari deb atalgan quyidagi formulalar bo'yicha topiladi:

$$x_1 = \frac{\Delta_1}{\Delta}, \qquad x_2 = \frac{\Delta_2}{\Delta}, \qquad x_3 = \frac{\Delta_3}{\Delta}.$$

Bu yerda Δ noma'lumlar oldidagi koeffitsiyentlardan tuzilgan kvadrat matritsa determinanti, Δ_1, Δ_2, Δ_3 lar asosiy matritsada mos ravishda 1, 2, 3-ustun elementlarini ozod hadlar bilan almashtirishdan hosil bo'lgan determinantlar. Shuni ta'kidlash kerakki, sistemada noma'lumlar va tenglamalar soni teng bo'lgan hollarda Kramer formulasini qo'llash maqsadga muvofiq.

Agar $\Delta \neq 0$ bo'lsa, sistema yagona yechimga ega bo'ladi.

Agar $\Delta=0$ bo'lib, Δ_1, Δ_2, Δ_3 lardan kamida bittasi noldan farqli bo'lsa sistema yechimga ega emas.

Agar $\Delta=0$ bo'lib, $\Delta_1=\Delta_2=\Delta_3=0$ bo'lsa, sistema aniqmas, cheksiz ko'p yechimga ega bo'ladi.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_3 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases} \qquad (1)$$

sistema uchun

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \qquad \Delta_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix},$$

$$\Delta_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}, \qquad \Delta_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$

Buni misollarda ko'ramiz:

1-misol.

a) $\begin{cases} x_1 + 2x_2 + \ \ x_3 = \ \ 8 \\ 3x_1 + 2x_2 + \ \ x_3 = 10 \\ 4x_1 + 3x_2 - 2x_3 = \ \ 4 \end{cases}$ sistemani Kramer formulasi bilan yeching.

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 4 & 3 & -2 \end{vmatrix} = \text{-4+8+9-8-3+12=14}$$

Δ≠0 bo'lgani uchun sistema aniq yagona yechim Kramer formulalari yordamida topiladi.

$$\Delta_1 = \begin{vmatrix} 8 & 2 & 1 \\ 10 & 2 & 1 \\ 4 & 3 & -2 \end{vmatrix} = \text{-32+8+30-8+40-24=14}$$

$$\Delta_2 = \begin{vmatrix} 1 & 8 & 1 \\ 3 & 10 & 1 \\ 4 & 4 & -2 \end{vmatrix} = \text{-20+32+12-40-4+48=28}$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 8 \\ 3 & 2 & 10 \\ 4 & 3 & 4 \end{vmatrix} = \text{8+80+72-64-24-30=42}$$

$$x_1 = \frac{14}{14} = 1, \quad x_2 = \frac{28}{14} = 2, \quad x_3 = \frac{42}{14} = 3. \qquad (1;2;3)$$

b) $\begin{cases} 4x_1 + 2x_2 + 3x_3 = -2 \\ 3x_1 + 8x_2 - \ \ x_3 = \ \ 8 \\ 9x_1 + \ \ x_2 + 8x_3 = \ \ 0 \end{cases}$ sistemani Kramer formulasi yordamida yeching.

$$\Delta = \begin{vmatrix} 4 & 2 & 3 \\ 2 & 8 & -1 \\ 9 & 1 & 8 \end{vmatrix} = \text{256+6-18-216-32+4=266-266=0}$$

Δ=0 Kramer teoremasiga ko'ra, sistema yoki aniqmas, yoki birgalikdamas. Δ_1 ni hisoblaymiz:

$$\Delta_1 = \begin{vmatrix} -2 & 2 & 3 \\ 8 & 8 & -1 \\ 0 & 1 & 8 \end{vmatrix} = -128+24-128-2= -234\neq 0$$

$\Delta=0$, $\Delta_1\neq 0$ bo'lgani uchun Kramer teoremasiga ko'ra sistema aniqlanmagan.

c) $\begin{cases} -2x_1 + x_2 - x_3 = 7 \\ 4x_1 + 5x_2 - 3x_3 = -5 \\ x_1 + 3x_2 - 2x_3 = 1 \end{cases}$ Kramer formulasiga ko'ra yeching.

$$\Delta = \begin{vmatrix} -2 & 1 & -1 \\ 4 & 5 & -3 \\ 1 & 3 & -2 \end{vmatrix} = 20-3-12+5+8-18=33-33=0$$

$\Delta=0$, demak sistema yoki aniqmas, yoki birgalikdamas. Δ_1, Δ_2, Δ_3 larni hisoblaymiz:

$$\Delta_1 = \begin{vmatrix} 7 & 1 & -1 \\ -5 & 5 & -3 \\ 1 & 3 & -2 \end{vmatrix} = -70+15-3+5-10+63=83-83=0$$

$$\Delta_2 = \begin{vmatrix} -2 & 7 & -1 \\ 4 & -5 & -3 \\ 1 & 1 & -2 \end{vmatrix} = -20-21-4-5+56-6=56-56=0$$

$$\Delta_3 = \begin{vmatrix} -2 & 1 & 7 \\ 4 & 5 & -5 \\ 1 & 3 & 1 \end{vmatrix} = -10-5+84-35-4-30=84-84=0$$

$\Delta=0$, $\Delta_1=\Delta_2=\Delta_3=0$ bo'lgani uchun sistema aniqmas, cheksiz ko'p yechimga ega.

Sistemani Gauss algoritmi bilan yechamiz:

$$\left(\begin{array}{ccc|c} -2 & 1 & -1 & 7 \\ 4 & 5 & -3 & -5 \\ 1 & 3 & -2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} -2 & 1 & -1 & 7 \\ 0 & 7 & -5 & 9 \\ 0 & \frac{7}{2} & -\frac{5}{2} & \frac{9}{2} \end{array}\right) \sim \left(\begin{array}{ccc|c} -2 & -1 & -1 & 7 \\ 0 & 7 & -5 & 9 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

berilgan tenglama $\begin{cases} -2x_1 + x_2 = x_3 + 7 \\ 4x_1 + 5x_2 = 3x_3 - 5 \\ x_3 \in R \end{cases}$ sistemaga teng kuchli.

Bu tenglamani Kramer formulasi bilan yechish mumkin.

$\Delta = \begin{vmatrix} -2 & 1 \\ 4 & 5 \end{vmatrix}$ = -10-4= -14

$\Delta_1 = \begin{vmatrix} x_3+7 & 1 \\ 3x_3-5 & 5 \end{vmatrix}$ $=5(x_3+7)-3x_3+5=5x_3+35-3x_3+5=2x_3+40=2(x_3+20)$

$\Delta_2 = \begin{vmatrix} -2 & x_3+7 \\ 4 & 3x_3-5 \end{vmatrix}$ $= -2(3x_3-5)-4(x_3+7) = -6x_3+10-4x_3-28 =$

$= -10x_3-18 = -2(5x_3+9)$

$$x_1 = \frac{2(x_3+20)}{-14} = -\frac{x_3+20}{7}, \qquad x_2 = \frac{-2(5x_3+9)}{-14} = \frac{5x_3+9}{7}$$

Sistema yechimi $\left(-\frac{x_3+20}{7};\ \frac{5x_3+9}{7};\ x_3\right)$ bo'ladi.

2. Chiziqli tenglamalar sistemasini teskari matritsa usulida yechish. Berilgan (1) sistemani

$$AX=B \qquad (2)$$

matritsa ko'rinishida yozib olamiz.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \qquad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \qquad B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

(2) tenglamani har ikki tomonini chapdan A^{-1} teskari matritsaga ko'paytiramiz.

$A^{-1} \cdot AX = A^{-1} \cdot B,$ $\qquad A^{-1} \cdot A = E$ bo'lgani uchun

$$X = A^{-1} \cdot B \qquad (3)$$

tenglik hosil bo'ladi.

(3) formula bilan topilgan X ustun matritsa sistemaning yechimi bo'ladi.

1-misolni a)-sini shu usul bilan yechamiz:

$$\begin{cases} x_1 + 2x_2 + x_3 = 8 \\ 3x_1 + 2x_2 + x_3 = 10 \\ 4x_1 + 3x_2 - 2x_3 = 4 \end{cases} \qquad \Delta = |A| = 14$$

$$A=\begin{pmatrix}1 & 2 & 1\\ 3 & 2 & 1\\ 4 & 3 & -2\end{pmatrix}$$ matritsa uchun teskari matritsa mavjud, chunki

$\Delta=|A|\neq 0.$ $$A^{-1}=\frac{1}{14}\begin{pmatrix}-7 & 7 & 0\\ 10 & -6 & 2\\ 1 & 5 & -4\end{pmatrix}$$

$$X=A^{-1}\cdot B=\frac{1}{14}\begin{pmatrix}-7 & 7 & 0\\ 10 & -6 & 2\\ 1 & 5 & -4\end{pmatrix}\cdot\begin{pmatrix}8\\ 10\\ 4\end{pmatrix}=\frac{1}{14}\begin{pmatrix}-56+70\\ 80-60+8\\ 8+50-16\end{pmatrix}=\begin{pmatrix}1\\ 2\\ 3\end{pmatrix};$$

Javob: $(1;2;3)$.

Mustaqil yechish uchun misollar:

Quyidagi tenglamalar sistemasini Kramer va teskari matritsa usulida yeching:

2. $\begin{cases} x+y-3z=-1\\ 2x-y+z=2\\ 3x+2y-4z=1\end{cases}$

3. $\begin{cases} x_1+3x_2-4x_3=-1\\ x_1-5x_2+x_3=7\\ 2x_1+x_2-3x_3=3\end{cases}$

4. $\begin{cases} 3x_1+x_2-x_3=2\\ 2x_1-3x_2+x_3=-1\\ x_1-x_2+2x_3=5\end{cases}$

5. $\begin{cases} 2x_1+2x_2-x_3+x_4=4\\ 4x_1+3x_2-x_3+2x_4=6\\ 8x_1+5x_2-3x_3+4x_4=12\\ 3x_1+3x_2-2x_3+2x_4=6\end{cases}$

6. $\begin{cases} 2x_1+5x_2+4x_3+x_4=20\\ x_1+3x_2+2x_3+x_4=11\\ 2x_1+10x_2+9x_3+7x_4=40\\ 3x_1+8x_2+9x_3+2x_4=37\end{cases}$

7. $\begin{cases} 2x-y-6z+3t+1=0\\ 7x-4y+2z-15t+32=0\\ x-2y-4z+9t-5=0\\ x-y-2z-6t+8=0\end{cases}$

8. $\begin{cases} 2x+y+4z+8t=-1\\ x+3y-6z+2t=3\\ 3x-2y+2z-2t=8\\ 2x-y-2z=4\end{cases}$

9. $\begin{cases} 3x+2y+z=5\\ x+y-z=0\\ 4x-y+5z=3\end{cases}$

10. $\begin{cases} 2x_1+x_2-x_3=5\\ 3x_1-x_2-2x_3=-5\\ 7x_1+x_2-x_3=10\end{cases}$

11. $\begin{cases} 2x-3y+z=2\\ x+5y-4z=-5\\ 4x+y-3z=-4\end{cases}$

12. $\begin{cases} 2x-4y+3z=1 \\ x-2y+4z=3 \\ 3x-y+5z=2 \end{cases}$

13. $\begin{cases} 2x-y+z=2 \\ 3x+2y+2z=-2 \\ x-2y+z=1 \end{cases}$

14. $\begin{cases} x+2y+3z=5 \\ 2x-y-z=1 \\ x+3y+4z=6 \end{cases}$

15. $\begin{cases} 2x_1-x_2+x_3=4 \\ 3x_1+2x_2-x_3=1 \\ x_1+x_2-2x_3=-3 \end{cases}$

16. $\begin{cases} x_1+x_2+x_3=2 \\ 3x_1+2x_2+2x_3=1 \\ 4x_1+3x_2+3x_3=4 \end{cases}$

17. $\begin{cases} x_1+x_2-2x_3=-7 \\ 3x_1-3x_2+x_3=12 \\ 5x_1-x_2-4x_3=-5 \end{cases}$

18. $\begin{cases} 2x_1+3x_2+11x_3+5x_4=2 \\ x_1+x_2+5x_3+2x_4=1 \\ 2x_1+x_2+3x_3+2x_4=-3 \\ 3x_1+x_2+3x_3+4x_4=-3 \end{cases}$

19. $\begin{cases} 3x_1+4x_2+x_3+2x_4+3=0 \\ 3x_1+5x_2+3x_3+5x_4+6=0 \\ 6x_1+8x_2+x_3+5x_4+8=0 \\ 3x_1+5x_2+3x_3+7x_4+8=0 \end{cases}$

20. $\begin{cases} 2x-y+3z=9 \\ 3x-5y+z=-4 \\ 4x-7y+z=5 \end{cases}$

21. $\begin{cases} 2x-5y+3z+t=5 \\ 3x-7y+3z-t=-1 \\ 5x-9y+6z+2t=7 \\ 4x-6y+3z+t=8 \end{cases}$

Javoblar:

2. (1; 1; 1)
3. (2; -1; 0)
4. (1; 2; 3)
5. $x_1=x_2=1,\ x_3=x_4=-1$
6. $x_1=1;\ x_2=x_3=2;\ x_4=0$
7. $x=-3,\ y=0,\ z=-0{,}5,\ t=2/3$
8. $x=2,\ y=-3,\ z=-1{,}5,\ t=0{,}5$
9. (-1; 3; 2)
10. (1; -2; -5)
11. (5; 6; 10)
12. (-1; 0; 1)
13. (2; -1; -3)
14. (1; -1; 2)
15. (1; 0; 2)
16. Sistema yechimga ega emas.
17. (1; -2; 3)
18. $x_1=-2,\ x_2=0,\ x_3=1,\ x_4=-1$
19. $x_1=-2,\ x_2=-2,\ x_3=1,\ x_4=-1$
20. Sistema yechimga ega emas.
21. Sistema yechimga ega emas.

6. CHIZIQLI TENGLAMALAR SISTEMASINI GAUSS VA GAUSS-JORDAN USULLARI BILAN YECHISH

1. Gaussning klassik usuli - bu berilgan sistemaning umumiy yechimini topishdan iborat bo'lib, bunda sistemaning tenglamalari ustida elementar almashtirishlar bajarib berilgan sistema trapetsiyali yoki uchburchakli ko'rinishga keltiriladi. So'ng oxirgi tenglamadan boshlab noma'lumlar ketma-ket topiladi.

1-misol. a) $\begin{cases} x_1+3x_2-4x_3=-4 \\ 3x_1-2x_2+x_3=11 \\ 4x_1-5x_2+x_3=9 \end{cases} \sim \begin{cases} x_1+2x_2-4x_3=-4 \\ -8x_2+13x_3=23 \\ -13x_2+17x_3=25 \end{cases} \sim \begin{cases} x_1+2x_2-4x_3=-4 \\ -8x_2+13x_3=23 \\ -\frac{33}{8}x_3=-\frac{99}{8} \end{cases}$

$x_3=3,\ x_2=2,\ x_1=4$ Javob: $(4;2;3)$.

2. Gauss-Jordan usuli noma'lumlarni ketma-ket yo'qotish Gauss usuli va teskari matritsa qurish Jordan algoritmiga asoslangan. Gauss-Jordan usuliga sxema ko'rinishida quyidagicha yoziladi: $(A|B)\sim(E|X)$.

$(A|B)$-asosiy matritsani ozod hadlar hisobiga kengaytirilgan matritsa.

E - birlik matritsa. X - tenglama yechimini ifodalovchi ustun matritsa.

b) $\begin{cases} x_1+x_2-6x_3-4x_4=6 \\ 3x_1-x_2-6x_3-4x_4=2 \\ 2x_1+3x_2+9x_3+2x_4=6 \\ 3x_1+2x_2+3x_3+8x_4=-7 \end{cases}$

Sistemani Gauss-Jordan usuli bilan yeching.

$$\left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 3 & -1 & -6 & -4 & 2 \\ 2 & 3 & 9 & 2 & 6 \\ 3 & 2 & 3 & 8 & -7 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 0 & -4 & 12 & 8 & -16 \\ 0 & 1 & 21 & 10 & -6 \\ 0 & -1 & 21 & 20 & -25 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & -3 & -2 & 4 \\ 0 & 1 & 21 & 10 & -6 \\ 0 & -1 & 21 & 20 & -25 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & -3 & -2 & 4 \\ 0 & 0 & 24 & 12 & -10 \\ 0 & 0 & 18 & 18 & -21 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & -3 & -2 & 4 \\ 0 & 0 & 1 & 1/2 & -5/12 \\ 0 & 0 & 18 & 18 & -21 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -3 & -2 & 2 \\ 0 & 1 & -3 & -2 & 4 \\ 0 & 0 & 1 & 1/2 & -5/12 \\ 0 & 0 & 0 & 9 & -27/2 \end{array}\right) \sim$$

$$\sim\left(\begin{array}{cccc|c}1&0&0&-1/2&3/4\\0&1&0&-1/2&11/4\\0&0&1&1/2&-5/12\\0&0&0&1&-3/2\end{array}\right)\sim\left(\begin{array}{cccc|c}1&0&0&0&0\\0&1&0&0&2\\0&0&1&0&1/3\\0&0&0&1&-3/2\end{array}\right)$$

Javob: (0; 2; 1/3; -3/2).

c) $\begin{cases}x_1-x_2+x_3=1\\x_1+x_2-2x_3=3\\2x_1+2x_2-4x_3=6\end{cases}$ Berilgan sistema birgalikda, chunki

$$r\begin{pmatrix}1&-1&1\\1&1&-2\\2&2&-4\end{pmatrix}=r\left(\begin{array}{ccc|c}1&-1&1&1\\1&1&-2&3\\2&2&-4&6\end{array}\right)$$

Sistema cheksiz ko'p yechimga ega, umumiy yechimni Gauss-Jordan usuli yordamida topamiz:

$$\left(\begin{array}{ccc|c}1&-1&1&1\\1&1&-2&3\\2&2&-4&6\end{array}\right)\to\left(\begin{array}{ccc|c}1&-1&1&1\\1&1&-2&3\\0&0&0&0\end{array}\right)\to\left(\begin{array}{ccc|c}1&-1&1&1\\1&1&-1&3\end{array}\right)\to\left(\begin{array}{ccc|c}1&-1&1&1\\0&2&-3&2\end{array}\right)\to$$

$$\to\left(\begin{array}{ccc|c}1&-1&1&1\\0&1&-3/2&1\end{array}\right)\to\left(\begin{array}{ccc|c}1&0&-1/2&2\\0&1&-3/2&1\end{array}\right)$$

$$\begin{cases}x_1-\frac{1}{2}x_3=2\\x_2-\frac{3}{2}x_3=1\end{cases}\Rightarrow\begin{cases}x_1=\frac{1}{2}x_3+2\\x_2=\frac{3}{2}x_3+1\end{cases}$$

Javob: $\left(\frac{1}{2}x_3+2;\ \frac{3}{2}x_3+1;\ x_3\right)\quad x_3\in R.$

Mustaqil yechish uchun misollar:

Quyidagi tenglamalar sistemasini Gauss, Gauss-Jordan usuli bilan yeching:

2. $\begin{cases}x_1-x_2+3x_3=3\\2x_1+3x_2-4x_3=-1\\3x_1+2x_2-x_3=2\end{cases}$ 3. $\begin{cases}x_1+2x_2+3x_3=0\\2x_1-3x_2-x_3=1\\3x_1+x_2+4x_3=-1\end{cases}$

4. $\begin{cases} x_1 - 3x_2 + 2x_3 + x_4 = 2 \\ 2x_1 + x_2 + 4x_3 + 3x_4 = 1 \\ x_1 + 5x_2 - x_3 + x_4 = -4 \\ 3x_1 - x_2 + 6x_3 + 5x_4 = 0 \end{cases}$

5. $\begin{cases} 2x_1 + x_2 + 3x_3 - 4x_4 = 3 \\ x_1 - 2x_2 + x_3 - 3x_4 = -1 \\ 3x_1 + 4x_2 - 5x_3 + x_4 = 4 \\ 2x_1 - 4x_2 + 2x_3 - 6x_4 = 5 \end{cases}$

6. $\begin{cases} x_1 - 2x_2 + x_3 = 4 \\ x_1 + 3x_2 + x_3 = 0 \end{cases}$

7. $\begin{cases} 3x_1 + x_2 = 0 \\ -x_1 + 2x_2 = 5 \\ 2x_1 - 4x_2 = 1 \end{cases}$

8. $\begin{cases} 3x_1 - 2x_2 - 5x_3 + x_4 = 3 \\ 2x_1 - 3x_2 + x_3 + 5x_4 = -3 \\ x_1 + 2x_2 - x_3 - 4x_4 = -1 \\ x_1 - x_2 - 4x_3 + 9x_4 = 22 \end{cases}$

9. $\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 15 \\ x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 35 \\ x_1 + 3x_2 + 6x_3 + 10x_4 + 15x_5 = 70 \\ x_1 + 4x_2 + 10x_3 + 20x_4 + 35x_5 = 126 \\ x_1 + 5x_2 + 15x_3 + 35x_4 + 70x_5 = 210 \end{cases}$

10. $\begin{cases} x + 2y + 3z = 5 \\ 2x - y - z = 1 \\ x + 3y + 4z = 6 \end{cases}$

11. $\begin{cases} 2x_1 - x_2 + x_3 - x_4 = 5 \\ x_1 + 2x_2 - 2x_3 + 3x_4 = -6 \\ 3x_1 + x_2 - x_3 + 2x_4 = -1 \end{cases}$

12. $\begin{cases} x_1 - 2x_2 - 5x_3 = 1 \\ 4x_1 + x_2 - 2x_3 = -3 \\ -x_1 + 3x_2 + 7x_3 = 2 \end{cases}$

13. $\begin{cases} -x_1 + 2x_2 - x_3 = 4 \\ 3x_1 + x_2 - 2x_3 = 1 \\ 4x_1 - x_2 + x_3 = -3 \end{cases}$

14. $\begin{cases} x_1 - 3x_2 = -5 \\ -x_1 + x_2 = 1 \\ 4x_1 - x_2 = 2 \end{cases}$

15. $\begin{cases} x_1 - x_2 - 3x_3 = 6 \\ -2x_1 + 2x_2 + 6x_3 = -9 \end{cases}$

16. $\begin{cases} x_1 + 2x_2 + x_3 = 8 \\ x_2 + 3x_3 + x_4 = 15 \\ 4x_1 + x_3 + x_4 = 11 \\ x_1 + x_2 + 5x_4 = 23 \end{cases}$

17. $\begin{cases} -x_1 + x_2 + x_3 - x_4 = -2 \\ x_1 + 2x_2 - 2x_3 - x_4 = -5 \\ 2x_1 - x_2 - 3x_3 + 2x_4 = -1 \\ x_1 + 2x_2 + 3x_3 - 6x_4 = -10 \end{cases}$

18. $\begin{cases} 4x_1 - 3x_2 + x_3 + 5x_4 - 7 = 0 \\ x_1 - 2x_2 - 2x_3 - 3x_4 - 3 = 0 \\ 3x_1 - x_2 + 2x_3 + 3x_4 + 1 = 0 \\ 2x_1 + 3x_2 + 2x_3 - 8x_4 + 7 = 0 \end{cases}$

19. $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 2 \\ 2x_1 + 3x_2 + 7x_3 + 10x_4 + 13x_5 = 12 \\ 3x_1 + 5x_2 + 11x_3 + 16x_4 + 21x_5 = 17 \\ 2x_1 - 7x_2 + 7x_3 + 7x_4 + 2x_5 = 57 \\ x_1 + 4x_2 + 5x_3 + 3x_4 + 10x_5 = 7 \end{cases}$

20. $\begin{cases} x_1 + x_2 - x_3 = 4 \\ 2x_1 + 4x_2 + x_3 = 9 \\ x_1 - x_2 + x_3 = -2 \\ 2x_1 + 5x_2 - 3x_3 = 15 \end{cases}$

Javoblar:

2. $\left\{\frac{8}{5}-x_3;\ 2x_3-\frac{7}{5};\ x_3\in R\right\}$

3. Sistema yechimga ega emas.

4. (1;0;2;-3)

5. Sistema yechimga ega emas.

6. $\left\{\frac{12}{5}x_3;\ -\frac{4}{5};\ x_3\in R\right\}$

7. Sistema yechimga ega emas.

8. $x_1=-1;\ x_2=3;\ x_3=-2;\ x_4=2.$

9. $x_1=5;\ x_2=4;\ x_3=3;\ x_4=2;\ x_5=1.$

10. (1;-1;2)

11. $\left\{\frac{4}{5}-\frac{1}{5}x_4;\ -\frac{17}{5}x_3-\frac{7}{5}x_4;\ x_3\in R;\ x_4\in R\right\}$

12. Sistema yechimga ega emas.

13. $\left\{-\frac{2}{7};\ \frac{13}{7};\ 0\right\}$

14. (1; 2)

15. Sistema yechimga ega emas.

16. (1; 2; 3; 4)

17. Yechimga ega emas.

18. (2; 1; -3; 1)

19. (3; -5; 4; -2; 1)

20. (1; 2; -1)

8. *N* – O'LCHOVLI ARIFMETIK FAZO. VEKTORLAR SISTEMASI. VEKTORNI VEKTORLAR SISTEMASI BO'YICHA YOYISH

1. n – o'lchovli arifmetik fazo deb, mumkin bo'lgan n ta $x_1, x_2, ...x_n$ haqiqiy sonlarning tartiblangan tizimlari to'plamiga aytiladi va R_n kabi belgilanadi.

$\boldsymbol{x}=(x_1, x_2, ...x_n)$ – R_n fazoning arifmetik vektori yoki nuqtasi deyiladi, $x_1, x_2, ...x_n$ haqiqiy sonlar $\boldsymbol{x}$ vektorning koordinatalari yoki komponentlari deyiladi. Komponentlar soni arifmetik vektor yoki nuqta o'lchovi hisoblanadi.

Oxyz koordinatalar sistemasida har qanday $\boldsymbol{x}$ vektorni $\bar{a} = a_x \bar{i} + a_y \bar{j} + a_z \bar{k}$ ko'rinishda yozish mumkin. Vektorning bu ko'rinishda yozilishi uning koordinata o'qlari bo'yicha yoyib yozishdir. a_x, a_y, a_z vektorning koordinata o'qlaridagi proeksiyalari. i, j, k - birlik vektorlar.

$\boldsymbol{a}$ vetor moduli yoki uzunligi $|\bar{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$ formula bo'yicha hisoblanadi.

$\vec{a}$ voktor yo`nalishi vektorning koordinata o'qlari *Ox, Oy, Oz* bilan hosil qilgan burchaklar bilan aniqlanadi, bu burchaklar kosinuslari:

$\cos\alpha = \dfrac{a_x}{|\bar{a}|}$, $\cos\beta = \dfrac{a_y}{|\bar{a}|}$, $\cos\gamma = \dfrac{a_z}{|\bar{a}|}$ formula bilan hisoblanadi,

bunda

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

tenglik o'rinli bo'ladi.

Uchlari $A(x_1; y_1; z_1)$, $B(x_2; y_2; z_2)$ nuqtalar bilan berilgan $\overline{AB}$ vektor koordinatasi

$$\overline{AB} = (x_2 - x_1; y_2 - y_1; z_2 - z_1)$$

ga teng bo'ladi.

$$\cos\alpha = \frac{x_2 - x_1}{|\overline{AB}|}; \quad \cos\beta = \frac{y_2 - y_1}{|\overline{AB}|}; \quad \cos\gamma = \frac{z_2 - z_1}{|\overline{AB}|}$$

1-misol. ABC uchburchakda, AN to'g'ri chiziq BAC burchak bissektrissasi hisoblanadi, N nuqta BC tomonda yotadi. Agar $|\overline{AB}| = \bar{b}$, $|\overline{AC}| = \bar{c}$ bo'lsa, $\overline{AN}$ vektor uzunligini toping.

Δ ABC dan $\overline{BC} = \bar{c} - \bar{b}$ uchburchakdan ichki burchak bissektrissasining xossasiga ko'ra $BN:NC=b:c$ yoki $|BN|:|BC|=b:(b+c)$; $\frac{BN}{c-b}=\frac{b}{b+c}$, bundan

$BN=\frac{b(\bar{c}-\bar{b})}{b+c}$

$\overline{AN}=\overline{AB}+\overline{BN}$ bo'lgani uchun $\overline{AN}=\bar{b}+\frac{b}{b+c}(\bar{c}-\bar{b})=\frac{b\bar{c}+c\bar{b}}{b+c}$ hosil bo'ladi.

2- misol. $A(1;\ 3;\ 2)$, $B(5;\ 8;-1)$ nuqtalar berilgan bo'lsa $\bar{a}=\overline{AB}$ vektorni toping.

AB vektorning proeksiyalari

$a_x=x_2-x_1=5-1=4;$ $\quad$ $a_y=y_2-y_1=8-3=5;$ $\quad$ $a_z=z_2-z_1=-1-2=-3$

formulalar bo'yicha hisoblanadi. Demak, $\overline{AB}=4\bar{i}+5\bar{j}-3\bar{k}$ ko'rinishda yoziladi.

n –o'lchovli arifmetik vektorlar ustida quyidagi chiziqli amallarni bajarish mumkin.

$\bar{x}=(x_1,\ x_2,\ \ldots..x_n)$, $\quad$ $\bar{y}=(y_1;y_2;\ldots..y_n)$ $\quad$ n- o'lchovli vektorlar va $\lambda>0$ haqiqiy son belirgan bo'lsin.

1) Vektorlarni qo'shish uchun mos koordinatalari qo'shiladi:

$$\bar{x}+\bar{y}=(x_1+y_1;\ x_2+y_2;\ \ldots.;\ x_n+y_n)$$

2) $\bar{x}$ vektorni λ songa ko'paytirish uchun berilgan vektorning har bir koordinatasini λ soniga ko'paytiriladi:

$$\lambda\bar{x}=(\lambda x_1;\ \lambda x_2;\ \ldots.;\ \lambda x_n)$$

3) $\bar{x}$; $\bar{y}$ vektorlarning skalyar ko'paytirish uchun mos koordinatalari ko'paytirilib, yig'indisi olinadi:

$$(x\cdot y)=x_1y_1+x_2y_2+\ldots+x_ny_n)$$

4) Vektorlar uzunliklari $|\overline{x}| = \sqrt{x_1^2 + x_2^2 + ... + x_n^2}$ formula bo'yicha topiladi.

5) $\overline{x}$; $\overline{y}$ vekorlar orasidagi burchak

$$\cos\varphi = \frac{(\overline{x}; \overline{y})}{|\overline{x}||\overline{y}|} = \frac{x_1 y_1 + x_2 y_2 + ... + x_n y_n}{\sqrt{x_1^2 + x_2^2 + ... + x_n^2}\sqrt{y_1^2 + y_2^2 + ... + y_n^2}}$$

formula bilan topiladi; *(φϵ[0;π])*

Misollar:

$\overline{x}$=(3; -4; 2;5), $\overline{y}$=(-1; 3 ; -7; 2) vektorlar va λ=2 haqiqiy son berilgan:

a) $3\overline{x}+2\overline{y}$ vektorni;

b) $\overline{x}*\overline{y}$ skalyar ko'paytmasini;

c) $\overline{x}$, $\overline{y}$ vetorlar orasidagi burchakni toping.

Yechish:

a) $3\overline{x}+2\overline{y}$=(9; -12; 6; 15)+(-2; 6; -14; 4)=(7; -6; -8; 19);

b) $\overline{x}*\overline{y}$=-3-12-14+10=-19;

c) $\cos\varphi = \frac{(\overline{x}; \overline{y})}{|\overline{x}||\overline{y}|}$; $|\overline{x}| = \sqrt{9+16+4+25} = \sqrt{54}$; $|\overline{y}| = \sqrt{1+9+49+4} = \sqrt{63}$;

$\cos\varphi = \frac{-19}{\sqrt{54}\sqrt{63}}$; $\varphi = -\arccos\frac{-19}{\sqrt{54}\sqrt{63}} = -\arccos\frac{-19}{9\sqrt{42}}$.

2. Vektorlar sistemasi. Vektorni vektorlar sistemasi bo'yicha yoyish.

n-o'lchovli *m* ta vektordan iborat vektorlar *n*-o'lchovli vektorlar sistemasini tashkil etadi.

$$\begin{cases} \overline{a_1}(a_{11}; a_{12}; ...... a_n) \\ \overline{a_2}(a_{21}; a_{22}; .... a_{2n}) \\ .... \quad .... \quad .... \\ \overline{a_m}(a_{m1}; a_{m2}; ..... a_{mn}) \end{cases}$$

$\overline{a_1}, \overline{a_2}, ..., \overline{a_n}$ vektorlar sistemasi va λ_1, λ_2, ….. λ_m haqiqiy sonlar berilgan bo'lsin.

$\lambda_1 \overline{a_1} + \lambda_2 \overline{a_2} + \ldots + \lambda_m \overline{a_m}$ vektorga $\overline{a_1}, \overline{a_2}, ..., \overline{a_n}$ vektorning λ_1, λ_2, ….. λ_m koeffitsientli chiziqli kombinatsiyasi deyiladi.

Vektorlar sistemasi va $\overline{b}(b_1; b_2; \ldots ; b_m)$ vektor berilgan bo'lsa $\overline{b}$ vektorni sistema vektorlari bo'yicha yoyish uchun $\sum_{j=1}^{m} \overline{a_j} x_j = \overline{b}$ chiziqli tenglamalar sistemasining yechimlaridan birini ko'rsatish yetarli.

Agar chiziqli tenglamalar sistemasi birgina yechimga ega bo'lsa, $\overline{b}$ vektor sistema vektorlari bo'yicha birgina usulda, agar cheksiz ko'p yechimga ega bo'lsa, cheksiz ko'p usulda yoyiladi, agar yechimga ega bo'lmasa $\overline{b}$ vektorni sistema vektorlari bo'yicha yoyib bo'lmaydi.

2- misol.

$\overline{b}(3;-1;4;5)$ vektorni

$\overline{a_1}(2;1;3;2)$, $\overline{a_2}(1;-2;4;-4)$, $\overline{a_3}(3;1;-5;2)$, $\overline{a_4}(-4;-3;1;-6)$ vektorlar sistemasi bo'yicha yoying.

Buning uchun $\overline{b} = \overline{a_1} x_1 + \overline{a_2} x_2 + \overline{a_3} x_3 + \overline{a_4} x_4$ vektor tenglama tuzib, uni Gauss - Jordan usulida yechmiz: $(A|B) \sim (E|X)$

$$(A|B) = \left(\begin{array}{cccc|c} 2 & 1 & 3 & -4 & 3 \\ 1 & -2 & 1 & -3 & -1 \\ 3 & 4 & -5 & 1 & 4 \\ 2 & -4 & 2 & -6 & 5 \end{array}\right)$$

berilgan vektorlar sistemasi koordinatalaridan tuzilgan matritsani ozod hadlar ustuni hisobiga kengaytirilgan matritsa. A matritsa o'rnida birlik matritsa hosil qilish uchun 2-satr elementlarini (-2) ga ko'paytirib 1-satrga, (-3) ga ko'paytirib 3-satrga, (-2) ga ko'paytirib 4-satrga qoshamiz:

$$\left(\begin{array}{cccc|c} 1 & 3 & 2 & -1 & 4 \\ 1 & -2 & 1 & -3 & -1 \\ 0 & 10 & -8 & 10 & 7 \\ 0 & 0 & 0 & 0 & 7 \end{array}\right)$$

bundan sistemaning yechimga ega emasligi ko'rinadi: $7 \neq 0$.

Demak, $\overline{b}$ vektorni $\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4}$ vektorlar sistemasi bo'yicha yoyish mumkin emas.

3-misol. $\overline{b}(5;1;6)$ vektorni

$\overline{a_1}(1;2;1), \overline{a_2}(2;-1;3), \overline{a_3}(3;-1;4)$ vektorlar sistemasi bo'yicha yoying.

Vektor tenglama tuzib Gauss - Jordan usulida yechamiz:

$$\overline{b} = \overline{a_1}x_1 + \overline{a_2}x_2 + \overline{a_3}x_3$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 2 & -1 & -1 & 1 \\ 1 & 3 & 4 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -5 & -7 & -9 \\ 0 & 1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 7/5 & 9/5 \\ 0 & 0 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 7/5 & 9/5 \\ 0 & 0 & -2/5 & -4/5 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 7/5 & 9/5 \\ 0 & 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right); \quad \begin{array}{l} x_1 = 1 \\ x_2 = -1 \\ x_3 = 2 \end{array} \quad \overline{b} = \overline{a_1} - \overline{a_2} + 2\overline{a_3}$$

Mustaqil yechish uchun masalalar:

1. $\overline{r} = \overline{OM} = 2\overline{i} + 3\overline{j} + 6\overline{k}$ vektor yasalsin va uning radius vektorining uzunligi hamda yo'nalishi aniqlansin. Vektorning uzunligi hamda yo'nalishi aniqlansin. $\mathrm{Cos}^2\alpha + \cos^2\beta + +\cos^2\gamma = 1$ formula bo'yicha tekshiring.

2. *M* nuqtaning radius vektori *0x* o'q bilan 45^0 va *0y* o'q bilan 60^0 burchak hosil etadi. Vektorning uzunligi *r*=6. Agar *M* ning applikatasi manfiy bo'lsa, uning koordinatalarini aniqlang va $\overline{OM} = \overline{r}$ vektorni $\overline{i}, \overline{j}, \overline{k}$ lar orqali ifodalang.

3. *x0y* tekislikda *A*(4;2), *B*(2;3), *C*(0;5) nuqtalar berilgan va $\overline{OA} = \overline{a}$, $\overline{OB} = \overline{b}$, $\overline{OC} = \overline{c}$ vektorlar yasalgan. $\overline{a}$ vektor $\overline{b}$ *va* $\overline{c}$ vektorlar bo'yicha topilsin.

4. Parallelogrammning ketma-ket uchta *A*(1; -2; 3), *B*(3; 2; 1), *C*(6; 4; 4) uchlari berilgan. Uning to'rtinchi uchini toping.

5. Uchlari *A*(2; -1; 3), *B*(1;1;1), *C*(0;0;5) nuqtalarda bo'lgan uchburchak *ABC* ning burchaklari aniqlansin.

6. $\overline{a}=2\overline{i}+\overline{j}$ va $\overline{b}=-\overline{2j}+\overline{k}$ vektorlarda yasalgan parallelogramm dioganallari orasidagi burchak topilsin.

7. $\overline{a}=\overline{i}+\overline{j}+2\overline{k}$ va $\overline{b}=\overline{i}-\overline{j}+4\overline{k}$ vektorlar berilgan. $\Pr_{\overline{b}}\overline{a}$ va $\Pr_{\overline{a}}\overline{b}$ aniqlansin.

8. 1) Agar *m* va *n* o'zaro 30^0 burchak tashkil etuvchi birlik vektorlar bo'lsa, $(m+n)^2$ hisoblansin.

2) Agar $|\overline{a}|=2\sqrt{2}$, $|\overline{b}|=4$ hamda $(\overline{a}\wedge\overline{b})=135^0$ bo'lsa, $(a-b)^2$ hisoblansin.

9. O'zaro komplanar $\overline{a}$, $\overline{b}$, $\overline{c}$ vektorlar berilgan bo'lib, $|\overline{a}|=3, |\overline{b}|=2, |\overline{c}|=5$ va $(\overline{a}\wedge\overline{b})=60^0$, $(\overline{b}\wedge\overline{c})=60^0$ bo'lsa, $\overline{u}=\overline{a}+\overline{b}-\overline{c}$ vektor yasalsin, $|\overline{u}|=\sqrt{(a+b+c)^2}$ formula bo'yicha uning moduli hisoblansin.

10. Teng yonli *OACB* trapetsiyada *M* va *N* nuqtalar mos ravishda *BC*=2, *AC*=2 tomonlarning o'rtalari. Trapetsiyaning o'tkir burchagi 60^0 ga teng. $\overrightarrow{OM}$ *va* $\overrightarrow{ON}$ vektorlar orasidagi burchak aniqlansin.

11. $\overline{a}(2;-1;3;4), \overline{b}(5;2;-2;6)$ vektorlar berilgan:

a) $2\overline{a}$, $5\overline{a}+3\overline{b}$, $\overline{a}-2\overline{b}$ vektorlarni;

b) $(\overline{a},\overline{b})$, $(3\overline{a}+b, \overline{a}-2\overline{b})$ skalyar ko'paytmalarini;

c) $\overline{a}$ *va* $\overline{b}$ vektor orasidagi burchakni toping.

Quyidagi $\overline{b}$ vektorlarni berilgan $\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4}$ vektorlar sistemasi bo'yicha yoyish mumkin yoki mumkin emasligini ko'rsating va yozing.

12. $\overline{b}=(-4;9); \overline{a_1}=(1;-3); \overline{a_2}=(2;-5);$

13. $\overline{b}=(8;-3;-10;10); \overline{a_1}=(1;0;4;3); \overline{a_2}=(1;1;-4;5);$
$\overline{a_3}=(1;-2;0;3); \overline{a_4}=(-2;3;1;4)$

14. $\overline{b}=(3;-1;4;5); \overline{a_1}=(2;1;3;2); \overline{a_2}=(1;-2;4;-4);$
$\overline{a_3}=(3;1;-5;2); \overline{a_4}=(-4;-3;1;-6)$

15. $\overline{b} = (9;-2;-3); \overline{a_1} = (1;-1;2); \overline{a_2} = (-5;-1;-4); \overline{a_3} = (4;1;5)$

16. λ ning qanday qiymatlarida $\overline{b} = (1;3;5)$ vektorni

$\overline{a_1} = (3;2;5), \overline{a_2} = (2;4;7), \overline{a_3} = (5;6;\lambda)$

vektorlar orqali yoyish mumkin?

17. $A(2;2;0)$ va $B(0;-2;5)$ nuqtalar berilgan. $\overline{AB} = \overline{u}$ vektor yasalsin, uning uzunligi va yo'nalishi aniqlansin.

18. 1) $(a+b)^2$;

2) $(a+b)^2-(a-b)^2$ ifodalardagi qavslar ochilsin va hosil bo'lgan formulalarning geometrik ma'nosi aniqlansin.

19. Agar $\overline{m}$ va $\overline{n}$ oralaridagi burchak 60^0 ga teng birlik vektorlar bo'lsa, $\overline{a} = 2\overline{m} + \overline{n}$ va $\overline{b} = \overline{m} - 2\overline{n}$ vektorlardan yasalgan parallelogramm dioganallarining uzunliklari aniqlansin.

20. Muntazam tetraedrning bir uchidan o'tkazilgan ikki tekis burchagining bissektrisalari orasidagi burchak aniqlansin.

21. $\overrightarrow{OA} = \overline{a}$ va $\overrightarrow{OB} = \overline{b}$ vektorlar berilgan. $|\overline{a}| = 2; |\overline{b}| = 4$ va $(a\wedge b)=60^0$. Uchburchak *OAB* ning *OM* medianasi bilan *OA* tomoni orasidagi bursak aniqlansin.

22. Tomonlari 6 va 4 *sm* bo'lgan to'g'ri to'rtburchak uchidan qarshi tomonlarini teng ikkiga bo'luvchi to'g'ri chiziqlar orasidagi burchaklar topilsin.

24. $\overline{m}$ va $\overline{n}$ lar o'zaro 120^0 burchak tashkil etuvchi birlik vektorlar bo'lsa, $\overline{a} = 2\overline{m} + 4\overline{n}$ va $\overline{b} = \overline{m} - \overline{n}$ vektorlar orasidagi burchak topilsin.

25. $\overline{a}(1;-3;2;0)$, $\overline{b}(4;-2;1;3)$, $\overline{c}(5;-3;2;1)$, $\overline{d}(1;2;2;-3)$ vektorlar uchun quyidagilarni hisoblang:

a) vektorlarning ortogonallarini aniqlang;

b) $(\overline{a}\,^\wedge\,\overline{b})$, $(\overline{b}\,^\wedge\,\overline{c})$, $(\overline{b}\,^\wedge\,\overline{d})$ larni hisoblang.

Javoblar:

1. r=7, cosα=2/7, cosβ=3/7, cosγ=6/7.

2. $M(3\sqrt{2};3;-3)$, $\overline{r} = 3(\sqrt{2}\vec{i} + \overline{j} - \overline{k})$

3. $\overline{a} = 2\overline{b} - 0{,}8\overline{c}$

4. Ko'rsatma *AD=BC* tenglikdan ular koordinatalarining tengligi (*x-1=6-3)* kelib chiqadi (4; 0; 6)

5. $B=C=45^0$

6. 90^0

7. $\Pr_{\overline{b}}\overline{a} = \frac{4\sqrt{2}}{3}$ $\Pr_{\overline{a}}\overline{b} = \frac{2}{3\sqrt{2}}$

8. 1) $2+\sqrt{3}$ 2) 40

9. 7

10. $\overline{OM} = \sqrt{(2+\overline{m})^2} = \sqrt{7}$; $\overline{ON} = \sqrt{(3\overline{m}+\overline{n})^2} = \sqrt{13}$;

$\cos\varphi = \frac{\overline{OM} * \overline{ON}}{|\overline{OM}| * |\overline{ON}|} = \frac{17}{2\sqrt{91}} = 0{,}891$; $\varphi=27^0$

11. *(4; -2; 6; 8); (25; 1; 9; 38); (-8; -5; 7; -8)*

12. $\overline{b} = 2\overline{a_1} - 3\overline{a_2}$

13. $\overline{b} = \overline{a_1} + 3\overline{a_2} - 2\overline{a_4}$

14. $\overline{b} = \overline{a_1} + 2\overline{a_2} - 3\overline{a_4}$

15. $\overline{b} = \overline{a_1} - 7\overline{a_2} - 7\overline{a_3}$

16. $\lambda \neq 12$ da

17. $\overline{u} = 3\sqrt{5}$; $\cos\alpha = -\frac{2}{3\sqrt{5}}$

18. $(a+b)^2 = a^2 + b^2 + 2ab\cos\varphi$ (kosinuslar teoremasi) $(a+b)^2-(a-b)^2=-a^2+2b^2$ (parallelogramm dioganallarining xossasi).

19. $\sqrt{7}$; $\sqrt{13}$

20. 5/6

21. $\cos\varphi = \frac{2}{\sqrt{7}}$

22. $\cos\varphi = 0{,}26\sqrt{10}$; $\varphi = 34^0 42'$

23. $D(-1;1;1)$; $\varphi = 120^0$

24. 120^0

25. a) ***c*** va ***d***

b) $\cos(a,b) = \frac{6}{\sqrt{105}}$; $\cos(b,c) = \frac{31}{3\cdot\sqrt{26}}$; $\cos(b;d) = -\frac{5}{6\cdot\sqrt{15}}$

9. CHIZIQLI BOG'LIQ VA CHIZIQLI ERKLI VEKTORLAR SISTEMASI

N o'lchovli m ta vektorlardan iborat vektorlar sistemasi berilgan bo'lsin.

$$\begin{cases} \overline{a_1}(a_{11}\, a_{12} \,.......\, a_{1n}) \\ \overline{a_2}(a_{21}\, a_{22} \,......\, a_{2n}) \\ -\;-\;-\;-\;-\;-\;-\;-\;-\;- \\ \overline{a_m}(a_{m1}\, a_{m2} \,.......\, a_{mn}) \end{cases} \qquad (1)$$

(1) Vektorlar sistemasi chiziqli erkli yoki chiziqli bog'liq ekanini aniqlash uchun berilgan vektorlar sistemasi vektorlaridan vektor tenglama tuzamiz:

$$\overline{a_1}x_1 + \overline{a_2}x_2 + ... + \overline{a_m}x_m = \overline{0} \qquad (2)$$

bu erda $\overline{0}$ - n o'lchovli nol vektor. (1) Tenglama m noma'lumli n ta bir jinsli chiziqli tenglamalar sistemasi. Bu sistema aniq bo'lib, yagona nol echimga ega bo'lsa, berilgan vektorlar sistemasi o'zaro chiziqli bog'liq bo'lmagan yoki chiziqli erkli vektorlar sistemasi bo'ladi.

Agar sistema aniqmas bo'lib, nol echimdan tashqari nolmas echimlarga ega bo'lsa, vektorlar sistemasi chiziqli bog`liq sistema bo'ladi; bunda $x_1, x_2, \ldots, x_m$ lardan kamida bittasi noldan farqli bo'lsa, $\overline{a_1}, \overline{a_2}, \ldots, \overline{a_m}$ lardan birini qolgan vektorlar orqali chiziqli ifodalash mumkin, bu esa sistema chiziqli bog'liq ekanini ko'rsatadi. (1) sistemaning chiziqli bog'liq yoki chiziqli erkli ekanini topish uchun vektorlar koordinatalaridan matritsa tuzamiz. Agar $r(A)=m$ bo'lsa, sistema chiziqli erkli, agar $r(A)<m$ bo'lsa, chiziqli bog'liq bo'ladi.

Misol-1. $\overline{a_1}(1;4;5)$, $\overline{a_2}(2;-1;1)$, $\overline{a_3}(-1;1;3)$ vektorlarning chiziqli bog'liq yoki chiziqli erkli ekanini aniqlang.

$A=\begin{pmatrix} 1 & 2 & -1 \\ 4 & -1 & 1 \\ 5 & 1 & 3 \end{pmatrix}$ matritsa rangini aniqlaymiz.

$$M=\begin{vmatrix} 1 & 2 & -1 \\ 4 & -1 & 1 \\ 5 & 1 & 3 \end{vmatrix}=-3+10-4-5-24-1=-27\neq 0$$

$$r(A)=3, \qquad r(A)=m=3.$$

Vektorlar sistemasi chiziqli erkli.

Misol-2. $\overline{a_1}(1;3;2)$, $\overline{a_2}(2;7;3)$, $\overline{a_3}(-1;2;-7)$ vektorlarning chiziqli bog'liq yoki chiziqli erkli ekanini aniqlang:

$$A=\begin{pmatrix} 1 & 2 & -1 \\ 3 & 7 & 2 \\ 2 & 3 & -7 \end{pmatrix}; \quad M=\begin{vmatrix} 1 & 2 & -1 \\ 3 & 7 & 2 \\ 2 & 3 & -7 \end{vmatrix}=-49+8-9+14+42-6=64-64=0$$

$$M_1=\begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix}=7-6=1\neq 0 \quad r(A)=2,$$

vektorlar soni $m=3$. $r(A)\neq m$. Vektorlar sistemasi chiziqli bogliq.

Mustaqil yechish uchun masalalar:

Vektorlar sistemasining chiziqli bog'liq yoki chiziqli bog'liq emasligini aniqlang:

1. $\overline{a_1}=(1;2;3)$, $\overline{a_2}=(3;6;7)$
2. $\overline{a_1}=(5;4;3)$, $\overline{a_2}=(3;3;2)$, $\overline{a_3}=(8;1;3)$
3. $\overline{a_1}=(4;-2;6)$, $\overline{a_2}=(6;-3;9)$
4. $\overline{a_1}=(4;-5;2;6)$, $\overline{a_2}=(2;-2;1;3)$, $\overline{a_3}=(6;-3;3;9)$, $\overline{a_4}=(4;-1;5;6)$
5. $\overline{a_1}=(2;-3;1)$, $\overline{a_2}=(3;-1;5)$, $\overline{a_3}=(1;-4;3)$
6. $\overline{a_1}=(1;0;0;2;5)$, $\overline{a_2}=(0;1;0;3;4)$, $\overline{a_3}=(0;0;1;4;7)$, $\overline{a_4}=(2;-3;4;11;12)$
7. $\overline{a_1}=(1;1;1)$, $\overline{a_2}=(0;1;1)$, $\overline{a_3}=(0;0;1)$
8. $\overline{a_1}=(3;2;1)$, $\overline{a_2}=(2;-3;0)$, $\overline{a_3}=(-3;-2;13)$
9. $\overline{a_1}=(3;2;1)$, $\overline{a_2}=(2;3;1)$, $\overline{a_3}=(-1;-4;-1)$.
10. $\overline{a}=(1;-3;4)$, $\overline{b}=(2;-1;0)$
11. $\overline{a_1}=(1;3;1;0)$, $\overline{a_2}=(-2;1;-3;-1)$, $\overline{a_3}=(4;0;5;1)$, $\overline{a_4}=(3;2;-1;-4)$

12. $\overline{x_1} = (-3;-1;5)$, $\overline{x_2} = (6;-3;15)$, $\overline{x_3} = (0;-5;25)$;

13. $\overline{a_1} = (1;1;1;1)$, $\overline{a_2} = (1;2;1;2)$, $\overline{a_3} = (3;2;-1;-4)$

14. $\overline{x_1} = (1;-1;1;0)$, $\overline{x_2} = (1;1;-1;1)$, $\overline{x_3} = (-1;3;-3;1)$

15. λ qanday qiymatlarida vektor $\overline{x} = (1;3;5)$ ni quyidagi $\overline{a_1}, \overline{a_2}, \overline{a_3}$ vektorlar orqali yoyish mumkin?

$$\overline{a_1} = (3;2;5),\quad \overline{a_2} = (2;4;7),\quad \overline{a_3} = (5;6;\lambda)$$

Quyidagi vektorlar sistemasini chiziqli bog'liq yoki chiziqli bog'liq emasligini aniqlang:

16. $\overline{a} = (-3;-2)$; $\overline{b} = (2;7)$

17. $\overline{a} = (1;-4;3)$; $\overline{b} = (2;-7;6)$

18. $\overline{a} = (1;-2;-3)$, $\overline{b} = (2;1;-1)$, $\overline{c} = (3;7;4)$

19. $\overline{a_1} = (2;0;-1;1)$, $\overline{a_2} = (3;8;1;5)$, $\overline{a_3} = (1;0;1;-3)$, $\overline{a_4} = (-1;0;1;-3)$

20. $\overline{a_1} = (1;2;0)$; $\overline{a_2} = (3;-1;1)$; $\overline{a_3} = (0;1;1)$

21. $\overline{x_1} = (1;1;1;1)$, $\overline{x_2} = (1,-1,-1;1)$, $\overline{x_3} = (1;-1;1;1)$, $\overline{x_4} = (1;1;-1;-1)$

22. $\overline{x_1} = (4;-5;2;6)$, $\overline{x_2} = (2;-2;1;3)$, $\overline{x_3} = (6;-3;3;9)$, $\overline{x_4} = (4;-1;5;6)$

$\overline{a_1} = (4;1;3;-2)$, $\overline{a_2} = (1;2;-3;2)$, $\overline{a_3} = (16;9;1;3)$, $\overline{a_4} = (0;1;2;3)$, $\overline{a_5} = (1;-1;15;0)$

23. $\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4}, \overline{a_5}$ vektorlar uchun quyidagi kombinatsiyani toping:

a) $\frac{1}{2}\overline{a_1} + 3\overline{a_2} - \frac{1}{2}\overline{a_4} + \overline{a_5}$ b) $\overline{a_2} - 5\overline{a_3} + \overline{a_4} + 2\overline{a_5}$

Tenglamadan x ni toping:

24. $2(\overline{a_1} - x) + 5(\overline{a_4} + x) = 0$ 25. $3(\overline{a_3} + 2x) - 2(\overline{a_5} - x) = 0$

Quyidagi $\overline{b}$ vektorni $\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4}$ vektorlar sistemasining chiziqli kombinatsiyasi ko'rinishida yoyish mumkin yoki mumkin emasligini ko'rsating:

26. $\overline{b_1} = (-5;-3;-2)$, $\overline{a_1} = (1;2;3)$, $\overline{a_2} = (0;1;-1)$, $\overline{a_3} = (3;4;-1)$

27. $\overline{b} = (5;1;-1;4)$, $\overline{a_1} = (1;2;0;3)$, $\overline{a_2} = (4;3;-2;1)$, $\overline{a_3} = (2;5;-4;-1)$, $\overline{a_4} = (1;6;-1;3)$

λ ning qanday qiymatlari vektor $\overline{b}$ ni quyidagi $\overline{a_1}, \overline{a_2}, \overline{a_3}$, vektorlar orqali chiziqli yoyish mumkin:

28. $\overline{a_1} = (2;3;5)$; $\overline{a_2} = (3;7;8)$; $\overline{a_3} = (1;-6;1)$, $\overline{b} = (7;-2;\lambda)$

29. $\overline{a_1} = (3;2;5)$; $\overline{a_2} = (2;4;7)$; $\overline{a_3} = (5;6;\lambda)$; $\overline{b} = (2;4;6)$

30. $\overline{a_1} = (3;0;0)$; $\overline{a_2} = (-1;1;0)$; $\overline{a_3} = (1;0;1)$; $\overline{b} = (-1;-1;\lambda)$

31. $\overline{a_1} = (0;1;0)$; $\overline{a_2} = (1;0;1)$; $\overline{a_3} = (0;4;0)$; $\overline{b} = (2;\lambda;-2)$?

Javoblar:

1. Chiziqli erkli.
2. Chiziqli bog'liq.
3. Chiziqli bog'liq.
4. Chiziqli bog'liq.
5. Chiziqli erkli.
6. Chiziqli erkli.
7. Chiziqli erkli.
8. Chiziqli erkli.
9. Chiziqli bog'liq.
10. Chiziqli bog'liq emas.
11. Chiziqli bog'liq.
12. Chiziqli bog'liq.
13. Chiziqli bog'liq emas.
14. Chiziqli bog'liq.
15. $\lambda \neq 12$.
16. Chiziqli bog'liq emas.
17. Chiziqli bog'liq emas.
18. Chiziqli bog'liq.
19. Chiziqli bog'liq emas.
20. Chiziqli bog'liq emas.
21. Chiziqli bog'liq emas.
22. Chiziqli bog'liq.
26. Yoyish mumkin.
27. Yoyish mumkin.
28. $\lambda=15$.
29. $\lambda \neq 12$.
30. $\lambda \in R$.
31. Hech qanday λ uchun.

10. VEKTORLAR SISTEMASINING RANGI VA BAZISI. VEKTORLAR SISTEMASINI ELEMENTAR ALMASHTIRISHLAR. KANONIK BAZIS

$\boldsymbol{a}_1, \ \boldsymbol{a}_2, \ldots, \boldsymbol{a}_n$ vektorlar sistemasi berilgan bo`lsin. Berilgan *vektorlar sistemasining bazisi* deb uning chiziqli bog’liq bo’lmagan shunday bir qismiga aytiladiki, bunda berilgan sistemaning har bir vektori bazis vektorlari orqali yoyilishi mumkin bo’ladi. Berilgan vektorlar sistemasining ixtiyoriy bazisi tarkibidagi vektorlar soniga uning *rangi* deyiladi.

1. Misol. Quyidagi vektorlar sistemasining bazislaridan birini quring va rangini aniqlang:

$\boldsymbol{a}_1(1;2;-1;3)$, $\boldsymbol{a}_2(0;3;4;1)$, $\boldsymbol{a}_3(-2;-1;6;-5)$, $\boldsymbol{a}_4(5;1;2;-4)$

Yechish: $\boldsymbol{a}_1x_1 + \boldsymbol{a}_2x_2 + \boldsymbol{a}_3x_3 + \boldsymbol{a}_4x_4 = \mathbf{0}$ vektor tenglama umumiy yechimini Gauss-Jordan usulida quramiz:

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 5 & 0 \\ 2 & 3 & -1 & 1 & 0 \\ -1 & 4 & 6 & 2 & 0 \\ 3 & 1 & -5 & -4 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & 5 & 0 \\ 0 & 3 & 3 & -9 & 0 \\ 0 & 4 & 4 & 7 & 0 \\ 0 & 1 & 1 & -19 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & 5 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -19 & 0 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{array}\right)$$

Yechilgan sistemadan x_1, x_2, x_4 - yechilgan noma’lumlar, x_3 esa erkin noma’lum ekanligi ko’rinib turibdi. Demak, berilgan vektorlar sistemasining bazisi $\boldsymbol{a}_1, \ \boldsymbol{a}_2$ va $\boldsymbol{a}_4$ vektorlar sistemasi bo’lib, sistemaning rangi bazisidagi vektorlar soni 3 ga teng.

Agar berilgan ikkita n o’lchovli $\boldsymbol{a}_1$ va $\boldsymbol{a}_2$ vektorlarning skalyar ko’paytmasi nolga teng bo’lsa, $\boldsymbol{a}_1$ va $\boldsymbol{a}_2$ vektorlar o’zaro *ortogonal vektorlar* deyiladi.

n o'lchovli nolmas vektorlardan tarkib topgan vektorlar sistemasi berilgan bo'lib, sistema vektorlarining har qanday ikki jufti o'zaro ortogonal bo'lsa, u holda sistemaga *ortogonal vektorlar sistemasi* deyiladi.

2. Misol. Quyidagi vektorlar sistemasi ortoganalmi?

$\boldsymbol{a_1}(0;5;-2)$, $\boldsymbol{a_2}(29;-2;-5)$, $\boldsymbol{a_3}(2;4;10)$

Yechish:

$(\boldsymbol{a_1} * \boldsymbol{a_2})=0-10+10=0$

$(\boldsymbol{a_1} * \boldsymbol{a_2})=0+20-20=0$

$(\boldsymbol{a_1} * \boldsymbol{a_2})=58-8-50=0$

Berilgan vektorlar sistemasi ortogonal vektolar sistemasi ekan.

Teng o`lchovli n ta $\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots \boldsymbol{a_n}$ chiziqli erkli vektorlar sistemasi ustida *ortoganal vektorlar sistemasini* qurish, ya'ni mos ravishda $\boldsymbol{b_1}, \boldsymbol{b_2}, \ldots \boldsymbol{b_k}$ ortogonal sistema bilan almashtirish mumkin. Buning uchun *Shmidt formulalari*dan foydalanamiz:

$\boldsymbol{b}_1=\boldsymbol{a}_1$

$$\mathrm{b_t} = a_t - \sum_{i=1}^{t-1} \frac{(b_i \cdot a_i)}{(b_i \cdot bi)} b_i \qquad t\epsilon\{2;3;\ldots k\}$$

3. Misol. $\boldsymbol{a_1}(1;1;1)$, $\boldsymbol{a_2}(0;1;1)$, $\boldsymbol{a_3}(0;0;1)$ vektorlar sistemasi ustida ortogonal sistema quring. rang $(\boldsymbol{a_1},\boldsymbol{a_2},\boldsymbol{a_3})=3$ chiziqli erkli sistema ekan.

$\boldsymbol{b_1}=\boldsymbol{a_1}(1;1;1)$

$$b_2 = a_2 - \frac{(b_1 \cdot a_3)}{(b_1 \cdot b_1)} b_1 = (0;1;1) - \frac{2}{3}(1;1;1) = \left(-\frac{2}{3};\frac{1}{3};\frac{1}{3}\right)$$

$$b_3 = a_3 - \frac{(b_1 \cdot a_3)}{(b_1 \cdot b_1)} b_1 = \frac{(b_2 \cdot a_3)}{(b_2 \cdot b_2)} b_2 = (0;0;1) - \frac{1}{3}(1;1;1) - \frac{1/3}{2/3}\left(-\frac{2}{3};\frac{1}{3};\frac{1}{3}\right) =$$

$$= \left(0;-\frac{1}{2};\frac{1}{2}\right)$$

Berilgan vektorlar sistemasi ustida qurilgan ortogonal sistema vektorlarini butun koordinatali vektorlarga aylantirib, (1;1;1); (-2;1;1); (0;-1;1) natijani olamiz.

Nolmas ***b*** *vektorning normallangan* yoki *birlik vektori* deb, $\frac{b}{|b|}$ vektorga aytiladi.

Har bir vektori normallangan, ya'ni birlik vektor ko'rinishiga keltirilgan ortoganal sistemaga *ortonormallangan vektorlar sistemasi* deyiladi.

3. Misol. Yuqoridagi misolga topilgan ortonormal $\boldsymbol{b_1}(1;1;1)$; $\boldsymbol{b_2}(-2;1;1)$; $\boldsymbol{b_3}(0;-1;1)$ sistemaning har bir vektorini birlik ko'rinishga keltiramiz.

$$\frac{b_1}{|b_1|}=\frac{1}{\sqrt{1^2+1^2+1^2}}(1;1;1)=\left(\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}}\right)$$

$$\frac{b_2}{|b_2|}=\frac{1}{\sqrt{(-2)^2+1^2+1^2}}(-2;1;1)=\left(-\frac{2}{\sqrt{6}};\frac{1}{\sqrt{6}};\frac{1}{\sqrt{6}}\right)$$

$$\frac{b_3}{|b_3|}=\frac{1}{\sqrt{0^2+(-1)^2+1^2}}(0;-1;1)=\left(0;-\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}}\right)$$

Mustaqil yechish uchun masalalar:

Quyida berilgan vektorlar sistemasining bazislaridan birini quring va ranglarini aniqlang:

1. $\boldsymbol{a_1}=(1;-2;-5)$, $\boldsymbol{a_2}=(3;4;-1)$, $\boldsymbol{a_3}=(2;-3;0)$

2. $\boldsymbol{a_1}=(1;1;-2;-5)$, $\boldsymbol{a_2}=(3;4;-1;2)$, $\boldsymbol{a_3}=(4;1;-2;3)$, $\boldsymbol{a_4}=(5;2;-3;1)$

3. $\boldsymbol{e_1}; \boldsymbol{e_2}; \boldsymbol{e_3}$ bazisda $\boldsymbol{a_1}=(1;1;0)$, $\boldsymbol{a_2}=(1;-1;1)$, $\boldsymbol{a_3}=(-3;5;6)$ vektorlar berilgan. $\boldsymbol{a_1}; \boldsymbol{a_2}; \boldsymbol{a_3}$ vektorlar bazisni tashkil qilishini ko`rsating.

4. $\boldsymbol{e_1}; \boldsymbol{e_2}; \boldsymbol{e_3}$ bazisda vektor $\boldsymbol{b}=(4;-4;5)$ berilgan. Shu vektorni quyidagi $\boldsymbol{a_1}; \boldsymbol{a_2}; \boldsymbol{a_3}$ bazisda ifodalang: $\boldsymbol{a_1}=(1;1;0)$, $\boldsymbol{a_2}=(1;-1;1)$, $\boldsymbol{a_3}=(-3;5;6)$

5. $\boldsymbol{e_1}; \boldsymbol{e_2}; \boldsymbol{e_3}$ bazisda berilgan $\boldsymbol{a}=(1;2;0)$, $\boldsymbol{b}=(3;-1;1)$, $\boldsymbol{c}=(0;1;1)$ vektorlar o`zlari bazis tashkil qilishini ko`rsating.

6. $\boldsymbol{e_1}; \boldsymbol{e_2}; \boldsymbol{e_3}$ bazisda quyidagi ***a, b, c*** vektorlar berilgan:
$\boldsymbol{a}=\boldsymbol{e_1}+\boldsymbol{e_2}+\boldsymbol{e}$, $\boldsymbol{b}=2\boldsymbol{e_2}+3\boldsymbol{e_3}$, $\boldsymbol{c}=\boldsymbol{e_2}+5\boldsymbol{e_3}$. ***a, b, c*** vektorlar bazis tashkil qilishini isbotlang. Vektor $\boldsymbol{b}=2\boldsymbol{e_1}-\boldsymbol{e_2}+\boldsymbol{e_3}$ ni ***a, b, c*** bazisdagi koordinatalarini toping.

Quyidagi vektorlar sistemasining bazislarini toping:

7.$\boldsymbol{a}_1$=(1;2;0;0); $\boldsymbol{a}_2$=(1;2;3;4); $\boldsymbol{a}_3$=(3;6;0;0);

8. $\boldsymbol{a}_1$=(1;2;3;4); $\boldsymbol{a}_2$=(2;3;4;5); $\boldsymbol{a}_3$=(3;4;5;6); $\boldsymbol{a}_4$=(4;5;6;7);

Berilgan vektorlar sistemasining rangi va barcha bazislari topilsin:

9. $\boldsymbol{a}_1$=(1;2;0;0); $\boldsymbol{a}_2$=(1;2;3;4); $\boldsymbol{a}_3$=(3;6;0;0);

10. $\boldsymbol{a}_1$=(1;2;3;4); $\boldsymbol{a}_2$=(2;3;4;5); $\boldsymbol{a}_3$=(3;4;5;6); $\boldsymbol{a}_4$=(4;5;6;7);

11. $\boldsymbol{a}_1$=(2;1;-3;1); $\boldsymbol{a}_2$=(4;2;-6;2); $\boldsymbol{a}_3$=(6;3;-9;3); $\boldsymbol{a}_4$=(1;1;1;1);

Vektorlar juftliklaridan o'zaro ortoganalmi:

12. $\boldsymbol{a}_1$(4;-5) va $\boldsymbol{a}_2$(1;0);

13. $\boldsymbol{a}_1$(4;1;2) va $\boldsymbol{a}_2$(-1;0;2);

14. $\boldsymbol{a}_1$(2;0;4;-1) va $\boldsymbol{a}_2$(1;2;3;4);

15. $\boldsymbol{a}_1$(1;3;2;-3) va $\boldsymbol{a}_2$(1;1;1;2)?

Quyida berilgan chiziqli erkli vektorlar sistemalari ustida ortogonal va ortonormallangan vektorlar sistemalari qurilsin:

16. $\boldsymbol{a}_1$(1;0) va $\boldsymbol{a}_2$(1;1)

17. $\boldsymbol{a}_1$(1;1;1;0), $\boldsymbol{a}_2$(0;1;1;1), $\boldsymbol{a}_3$(0;0;1;1)

Quyida berilgan vektorlar sistemasining rangi va bazislari topilsin:

18. $\boldsymbol{a}_1$=(5;2;-3;1); $\boldsymbol{a}_2$=(4;1;-2;3); $\boldsymbol{a}_3$=(1;1;-1;2); $\boldsymbol{a}_4$=(3;4;-1;2)

19. $\boldsymbol{a}_1$=(2;-1;3;5); $\boldsymbol{a}_2$=(4;-3;1;3); $\boldsymbol{a}_3$=(3;-2;3;4); $\boldsymbol{a}_4$=(4;-1;15;17); $\boldsymbol{a}_5$=(7;-6; -7;0)

20. $\boldsymbol{a}_1$=(2;1;-3;1); $\boldsymbol{a}_2$=(4;2;-6;2); $\boldsymbol{a}_3$=(6;3;-9;3); $\boldsymbol{a}_4$=(1;1;1;1)

21. $\boldsymbol{a}_1$=(1;2;3); $\boldsymbol{a}_2$=(2;3;4); $\boldsymbol{a}_3$=(3;2;3); $\boldsymbol{a}_4$=(4;3;4) $\boldsymbol{a}_5$=(1;1;1)

22. $\boldsymbol{a}_1$=(5;2;-3;1); $\boldsymbol{a}_2$=(4;1;-2;3); $\boldsymbol{a}_3$=(1;1;-1;-2); $\boldsymbol{a}_4$=(3;4;-1;2)

23. $\boldsymbol{a}_1$=(2;-1;3;5); $\boldsymbol{a}_2$=(4;-3;1;3); $\boldsymbol{a}_3$=(3;-2;3;4); $\boldsymbol{a}_4$=(4;-1;15;17); $\boldsymbol{a}_5$=(-7;-6;-7;0)

Quyida berilgan chiziqli erkli vektorlar sistemalari ustida ortogonal va ortonormallangan vektorlar sistemalari qurilsin:

24. $\boldsymbol{a}_1$(1;1), $\boldsymbol{a}_2$(0;2)

25. $\boldsymbol{a}_1$(1;0;1;0), $\boldsymbol{a}_2$(0;1;1;1), $\boldsymbol{a}_3$(1;1;0;1)

26. $\boldsymbol{a}_1(1;1;1;1)$, $\boldsymbol{a}_2(1;1;1;0)$, $\boldsymbol{a}_3(1;0;1;1)$

Javoblar:

1. Bazisi $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{a}_3$, rangi 3 2. Bazisi $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{a}_3$, rangi 3

4. $\boldsymbol{b}=0.5\boldsymbol{a}_1+2\boldsymbol{a}_2-0.5\boldsymbol{a}_3$ 6. (2;-2;1)

7. a) $\boldsymbol{a}_1$, $\boldsymbol{a}_2$; b) $\boldsymbol{a}_2, \boldsymbol{a}_3$

8. Ixtiyoriy ikkita vektor bazis tashkil etadi.

9. $r=2$; $(\boldsymbol{a}_1\ \boldsymbol{a}_2)$, $(\boldsymbol{a}_2\ \boldsymbol{a}_3)$ 10. $r=2$; $(\boldsymbol{a}_1\ \boldsymbol{a}_2)$, $(\boldsymbol{a}_1\ \boldsymbol{a}_3)$,$(\boldsymbol{a}_1\ \boldsymbol{a}_4)$

11. $r=2$; $(\boldsymbol{a}_1\ \boldsymbol{a}_4)$, $(\boldsymbol{a}_2\ \boldsymbol{a}_4)$,$(\boldsymbol{a}_3\ \boldsymbol{a}_4)$ 12. Ortogonal emas.

13. Ortogonal. 14. Ortogonal emas.

15. Ortogonal.

16. Ortogonal va ortonormal vektorlar: $\boldsymbol{b}_1(1;0)$, $\boldsymbol{b}_2(0;1)$.

17. $\boldsymbol{b}_1(1;1;1;0)$, $\boldsymbol{b}_2(-2;1;1;3)$, $\boldsymbol{b}_3(1;-3;2;1)$

$$\frac{b_1}{|b_1|}=\left(\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}};0\right),\quad \frac{b_2}{|b_2|}=\left(-\frac{2}{\sqrt{15}};\frac{1}{\sqrt{15}};\frac{1}{\sqrt{15}};\frac{3}{\sqrt{15}}\right),$$

$$\frac{b_3}{|b_3|}=\left(\frac{1}{\sqrt{15}};-\frac{3}{\sqrt{15}};\frac{2}{\sqrt{15}};\frac{1}{\sqrt{15}}\right)$$

18. $\boldsymbol{a}_2$, $\boldsymbol{a}_3$, $\boldsymbol{a}_4$ - *bazis*, $r=3$ 19. $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{a}_3$ – *bazis*, $r=3$

20. a) $\boldsymbol{a}_1, \boldsymbol{a}_4$ b) $\boldsymbol{a}_2, \boldsymbol{a}_4$ c) $\boldsymbol{a}_3, \boldsymbol{a}_4$

21. $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{a}_5$ va $\boldsymbol{a}_3$, $\boldsymbol{a}_4$, $\boldsymbol{a}_5$ lardan tashqari ixtiyoriy uchta vector bazisni tashkil qiladi.

22. Ikkita bazis. 23. $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{a}_3$ bazis tashkil qiladi.

24. $\boldsymbol{b}_1=(1;1)$; $\boldsymbol{b}_2=(-1;1)$; $\frac{b_1}{|b_1|}=\left(\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}}\right)$, $\frac{b_2}{|b_2|}=\left(-\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}}\right)$

25. $\boldsymbol{b}_1=(1;0;1;0)$; $\boldsymbol{b}_2=(-1;2;1;2)$; $\boldsymbol{b}_3=(2;1;-2;1)$

$$\frac{b_1}{|b_1|}=\left(\frac{1}{\sqrt{2}};0;\frac{1}{\sqrt{2}};0\right),\quad \frac{b_2}{|b_2|}=\left(-\frac{1}{\sqrt{10}};\frac{2}{\sqrt{10}};\frac{1}{\sqrt{10}};\frac{2}{\sqrt{10}}\right),\quad \frac{b_3}{|b_3|}=\left(\frac{2}{\sqrt{10}};\frac{1}{\sqrt{10}};-\frac{2}{\sqrt{10}};1\right),$$

26. $\boldsymbol{b}_1=(1;1;1;1)$; $\boldsymbol{b}_2=(1;1;1;-3)$; $\boldsymbol{b}_3=(1;-2;1;0)$

$$\frac{b_1}{|b_1|}=\left(\frac{1}{2};\frac{1}{2};\frac{1}{2};\frac{1}{2}\right),\quad \frac{b_2}{|b_2|}=\left(\frac{1}{\sqrt{12}};\frac{1}{\sqrt{12}};\frac{1}{\sqrt{12}};-\frac{3}{\sqrt{12}}\right),\quad \frac{b_3}{|b_3|}=\left(\frac{1}{\sqrt{6}};-\frac{2}{\sqrt{6}};\frac{1}{\sqrt{6}};0\right),$$

11. VEKTOR KO`RINISHIDA YOZILGAN CHIZIQLI TENGLAMALAR SISTEMASINING BIRGALIKDALIK VA ANIQLIK SHARTLARI. FUNDAMENTAL YECHIMLAR

m ta noma'lumli *n* ta chiziqli *bir jinsli tenglamalar sistemasi* vektor shaklda berilgan bo'lsin:

$$\mathbf{a}_1x_1+\mathbf{a}_2x_2+\ldots+\mathbf{a}_mx_m=\theta$$

$rang(\mathbf{a}_1\mathbf{a}_2,\ldots,\mathbf{a}_m)=rang(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_m,\mathbf{b})$ bo'lgani uchun sistema har doim *birgalikda*. $Rang(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_m)=m$ munosabat o'rinli bo'lsa, sistema *aniq* va yagona nol yechimga ega.

$Rang(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_m)<m$ munosabat o'rinli bo'lsa, sistema *aniqmas* **v**a yechimdan tashqari nolmas yechimlarga ham ega bo'ladi. Ushbu holda, har bir nolmas yechim *m* o'lchovli vektor sifatida qaralishi mumkin.

Bir jinsli chiziqli tenglamalar sistemasining fundamental yechimlari sistemasi yoki tizimi deb, uning chiziqli bog'liq bo'lmagan nolmas $\mathbf{F}_1, \mathbf{F}_2,\ldots,\mathbf{F}_k$ yechimlariga aytiladiki**,** sistemaning har bir yechimi ushbu yechimlarning chiziqli kombinatsiyasi ko'rinishida aniqlanishi mumkin.

Agar $rang(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_m)=r<m$ bo'lsa, sistema o'zining fundamental yechimlari tizimi mavjudligi bilan harekterlanadi va tizim har bir *m* o'lchovli *m-r* ta nolmas vektorlardan tarkib topadi.

Bir jinsli sistemaning fundamental yechimlari tizimi quyidagicha quriladi:

1. Bir jinsli sistemaning umumiy yechimi quriladi;
2. *m-r* o'lchovli *m-r* ta vektorlardan iborat chiziqli erkli vektorlar siatemasi, masalan: $\mathbf{e}_1(1;0;\ldots;0)$, $\mathbf{e}_2(0;1;0;\ldots;0)$,…, $\mathbf{e}_{m-r}(0;0;\ldots;1)$ tanlanadi;
3. Umumiy yechim erkli noma'lumlari o'rniga $\mathbf{e}_1$ vektor mos koordinatalarini qo'yib, bazis noma'lumlar aniqlanadi va mos ravishda $\mathbf{F}_1$ fundamental yechim quriladi. Shuningdek, $\mathbf{e}_2, \mathbf{e}_3, \ldots, \mathbf{e}_{m-r}$ vektorlardan foydalanib, mos ravishda $\mathbf{F}_2, \mathbf{F}_3, \ldots, \mathbf{F}_{m-r}$ fundamental yechimlar quriladi.

1. Misol. Bir jinsli sistemaning fundamental yechimlari tizimidan birini quring va uning umumiy yechimini vektor shaklda aniqlang:

$$\begin{cases} 4x_1 + 7x_2 + 2x_3 + 3x_4 = 0 \\ x_1 + 3x_2 - x_3 + 2x_4 = 0 \\ 2x_1 + x_2 + 4x_3 - x_4 = 0 \end{cases}$$

Sistemaning umumiy yechimini Gayss-Jordan usulida quramiz:

$$\begin{pmatrix} 4 & 7 & 2 & 3 & | & 0 \\ 1 & 3 & -1 & 2 & | & 0 \\ 2 & 1 & 4 & -1 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & -5 & 6 & -5 & | & 0 \\ 1 & 3 & -1 & 2 & | & 0 \\ 0 & -5 & 6 & -5 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & -1{,}2 & 1 & | & 0 \\ 1 & 0 & -4{,}6 & -1 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

m=4, r=2 bo'lgani uchun m-r=2 ta chiziqli erkli $\boldsymbol{e}_1$(1;0) va $\boldsymbol{e}_2$(0;1) sistemani tanlaymiz. $\boldsymbol{e}_1$(1;0) vektor koordinatalarini umumiy yechimning mos erkli nomalumlari o'rniga qo'yib, bazis nomalumlarni aniqlaymiz va $\boldsymbol{F}_1$(4,6;1,2;1;0) fundamental echimni quramiz. $\boldsymbol{e}_2$(0;1) vector yordamida $\boldsymbol{F}_2$(1;-1;0;1) fundamental yechimni quramiz. Boshqacha qilib aytganda kengaytirilgan matritsadagi koeffitsiyentlarni sistemaga qo'yamiz:

$$\begin{cases} x_2 - 1{,}2x_3 + x_4 = 0 \\ x_1 \quad - 4{,}6x_3 - x_4 = 0 \end{cases} \qquad \begin{cases} x_1 = 4{,}6x_3 + x_4 \\ x_2 = 1{,}2x_3 - x_4 \\ x_3 = x_3 \\ x_4 = x_4 \end{cases}$$

Fundamental yechimlar $\boldsymbol{F}_1$(4,6;1,2;1;0) va $\boldsymbol{F}_2$(1;-1;0;1) quriladi.

Umumiy yechimni tuzamiz:

$$\boldsymbol{X} = \lambda_1 \boldsymbol{F}_1 + \lambda_2 \boldsymbol{F}_2 = \lambda_1 \begin{pmatrix} 4{,}6 \\ 1{,}2 \\ 1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

Bu yerda λ_1 va λ_2 lar ixtiyoriy haqiqiy sonlar.

m ta nomalumli n ta chiziqli bir jinsli bo'lmagan tenglamalar sistemasi vektor shaklda berilgan bo'lsin:

$\boldsymbol{a}_1 x_1 + \boldsymbol{a}_2 x_2 + \ldots + \boldsymbol{a}_m x_m = \boldsymbol{b}$ $\quad (\boldsymbol{b} \neq 0)$

Sistemaning umumiy yechimini vektor shaklda yozish mumkin:

$\boldsymbol{X}=\boldsymbol{F_0}+\lambda_1 \boldsymbol{F_1}+\lambda_2 \boldsymbol{F_2}+\ldots+\lambda_{m-r} \boldsymbol{F_{m-r}}$

Bu yerda $\boldsymbol{F_0}$ - bir jinslimas sistemaning xususiy yechimlaridan biri, $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, ..., $\boldsymbol{F_{m-r}}$ - berilgan sistemaga mos ravishdagi

$\boldsymbol{a_1}x_1+\boldsymbol{a_2}x_2+\ldots+\boldsymbol{a_m}x_m=\theta$

bir jinsli tenglamalar sistemasining fundamental yechimlari tizimi, $\lambda_1, \lambda_2, \ldots, \lambda_{m-r}$ -ixtiyoriy haqiqiy sonlar.

2. Misol. Berilgan sistema umumiy yechimini vektor shaklda quring:

$$\begin{cases} 4x_1+7x_2+2x_3+3x_4=16 \\ x_1+3x_2-x_3+2x_4=3 \\ 2x_1+x_2+4x_3-x_4=2 \end{cases}$$

$$\begin{pmatrix} 4 & 7 & 2 & 3 & | & 16 \\ 1 & 3 & -1 & 2 & | & 3 \\ 2 & 1 & 4 & -1 & | & 2 \end{pmatrix} \sim \begin{pmatrix} 0 & -5 & 6 & -5 & | & 4 \\ 1 & 3 & -1 & 2 & | & 3 \\ 0 & -5 & 6 & -5 & | & 4 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & -1{,}2 & 1 & | & -0{,}8 \\ 1 & 0 & -4{,}6 & -1 & | & 5{,}4 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$\boldsymbol{F_0}(5{,}4; -0{,}8; 0; 0)$ sistemaning xususiy yechimlaridan birini qurdik.

Sistema umumiy yechimi vektor shaklini yozamiz:

$$\boldsymbol{X}=\boldsymbol{F_0}+\lambda_1 \boldsymbol{F_1}+\lambda_2 \boldsymbol{F_2}=\begin{pmatrix} 5{,}4 \\ -0{,}8 \\ 0 \\ 0 \end{pmatrix}+\lambda_1\begin{pmatrix} 4{,}6 \\ 1{,}2 \\ 1 \\ 0 \end{pmatrix}+\lambda_2\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

bu yerda λ_1 va λ_2 lar ixtiyoriy haqiqiy sonlar.

Mustaqil yechish uchun masalalar:

Bir jinsli tenglamalar sistemasini yeching:

1. $\begin{cases} x_1+2x_2-x_3-x_4=0 \\ -2x_1-4x_2+2x_3-2x_4=0 \\ 3x_1+6x_2-3x_3-3x_4=0 \end{cases}$

2. $\begin{cases} x_1-2x_2+x_3=0 \\ x_1+2x_2-2x_3=0 \\ 3x_1+2x_2-3x_3=0 \end{cases}$

3. $\begin{cases} x_1-7x_2+5x_3-3x_4=0 \\ 2x_1-3x_2+7x_3-x_4=0 \\ x_1-x_2+x_3-2x_4=0 \\ 5x_2+x_3=0 \end{cases}$

4. $\begin{cases} x_1+2x_2+4x_3-3x_4=0 \\ 3x_1+5x_2+6x_3-4x_4=0 \\ 4x_1+5x_2-2x_3-3x_4=0 \\ 3x_1+8x_2-24x_3-19x_4=0 \end{cases}$

Bir jinsli bo'lmagan chiziqli tenglamalar sistemalarining umumiy yechimini toping:

5. $\begin{cases} x_1 - 2x_2 + 3x_3 + x_4 = 1 \\ 2x_1 - 3x_2 - x_3 - x_4 = -4 \end{cases}$

6. $\begin{cases} 2x_1 - x_2 + x_3 + x_4 = 1 \\ x_2 - x_3 + 2x_4 = 2 \\ 2x_2 - 2x_3 + 3x_4 = 3 \end{cases}$

7. $\begin{cases} x_1 + 2x_2 - x_3 = 5 \\ 2x_1 - x_2 - 3x_3 = 4 \end{cases}$

8. $\begin{cases} 3x_1 + x_2 - x_3 - 2x_4 = -4 \\ x_1 - x_2 - x_3 + 2x_4 = 1 \end{cases}$

9. $\begin{cases} x_1 + x_2 + x_3 + x_4 = 2 \\ 2x_2 - 2x_3 + 2x_4 = 2 \\ x_1 - x_3 + x_4 = 2 \end{cases}$

Sistemani yeching:

10. $\begin{cases} 3x - 2y - z = 0 \\ 2x - y + 3z = 0 \\ -3y - 4z = 0 \end{cases}$

11. $\begin{cases} 3x + 2y - z = 0 \\ 2x - y + 3z = 0 \\ x + y - z = 0 \end{cases}$

12. $\begin{cases} 3x_1 - 2x_2 + 3x_3 + 3x_4 = 0 \\ 3x_1 - 2x_2 - x_3 + x_4 = 0 \\ x_1 - x_2 + 2x_3 + 5x_4 = 0 \end{cases}$

13. $\begin{cases} x_1 - 2x_2 + 4x_3 - 3x_4 = 0 \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 = 0 \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 = 0 \\ 3x_1 + 8x_2 + 24x_3 - 19x_4 = 0 \end{cases}$

Sistemalarni fundamental yechimlarini va umumiy yechimini toping:

14. $\begin{cases} 3x_1 - x_2 + 2x_3 + 3x_4 = 18 \\ -x_1 - x_2 + 2x_4 = 0 \\ x_1 + x_2 + 3x_3 - 2x_4 = 0 \end{cases}$

15. $\begin{cases} 3x + 5y + 2z = 0 \\ 5x + 2y + 3z = 0 \end{cases}$

16. $\begin{cases} 2x_1 + x_2 - 4x_3 - x_4 + x_5 = 0 \\ x_1 + 2x_2 + x_3 + 2x_4 + x_5 = 0 \\ 3x_1 - x_2 - 5x_3 + x_4 + 2x_5 = 0 \end{cases}$

17. $\begin{cases} x_1 - 3x_2 - x_3 + 4x_4 - x_5 = 7 \\ 2x_1 - x_2 - 3x_3 + x_4 + 4x_5 = -3 \\ 3x_1 - 2x_2 - 2x_3 + 5x_4 + 3x_5 = 4 \end{cases}$

18. $\begin{cases} x_1 + 2x_2 + 3x_3 = 2 \\ x_1 - x_2 + x_3 = 0 \\ x_1 + 3x_2 - x_3 = -2 \\ 3x_1 + 4x_2 + 3x_3 = 0 \end{cases}$

Javoblar:

1. (-2;1;0;0), yoki (1;0;1;0), yoki (1;0;0;1)
2. $(\frac{1}{2};\frac{3}{4};1)$
3. (0;0;0;0)
4. (8;-6;1;0), (-7;5;0;1)
5. $\boldsymbol{F_0}$(-11;-6;0;0), $\boldsymbol{F_1}$(11;7;1;0), $\boldsymbol{F_2}$(5;3;0;1)
6. $\boldsymbol{F_1}$=(-1;1;1;1), $\boldsymbol{F_0}$=(0;0;0;1)
7. $\boldsymbol{F_0}=(\frac{13}{5};\frac{6}{5};0)$, $\boldsymbol{F_1}=(\frac{7}{5};-\frac{1}{5};1)$
8. $\boldsymbol{F_0}=(1;\frac{7}{2};0;0)$, $\boldsymbol{F_1}=(-\frac{5}{2};\frac{9}{2};1;0)$, $\boldsymbol{F_2}=(-\frac{1}{2};-\frac{1}{2};0;1)$

10. $\boldsymbol{F_0}$=(0;0;0), $\boldsymbol{F_1}=(-\frac{5}{7};\frac{11}{7};1)$
11. (0;0;0)
12. $\boldsymbol{F_1}=(\frac{13}{2};\frac{7}{3};\frac{1}{2};1)$
13. $\boldsymbol{F_1}$=(8;-6;1;0) , $\boldsymbol{F_2}$=(13;-5;0;1)
14. $\boldsymbol{F_0}=(\frac{13}{3};-\frac{13}{3};\frac{1}{3};0)$, $\boldsymbol{F_1}=(\frac{49}{12};-\frac{25}{12};\frac{1}{3};1)$
15. $\boldsymbol{F_1}=(-\frac{1}{2};-\frac{1}{4};1)$
16. $\boldsymbol{F_1}$=(0;1;1;0) , $\boldsymbol{F_2}=(-\frac{3}{5};\frac{1}{5};0;1)$
17. $\boldsymbol{F_0}=(-\frac{2}{7};-\frac{17}{7};0;0;0)$, $\boldsymbol{F_1}=(-\frac{6}{7};-\frac{12}{7};1;0;0)$

    $\boldsymbol{F_2}=(-\frac{9}{7};-\frac{12}{7};0;1;0)$, $\boldsymbol{F_3}=(-\frac{19}{7};-\frac{23}{7};0;0;1)$
18. (-1;0;1) –yagona yechim.

12. TEKISLIKDAGI TO'G'RI CHIZIQ TENGLAMALARI. TO'G'RI CHIZIQ NORMAL TENGLAMASI. NUQTADAN CHIZIQQACHA BO'LGAN MASOFA

1^0. *Tekislikdagi* $A(x_1;\, y_1)$ va $B(x_2;\, y_2)$ nuqtalar orasidagi masofa:

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \qquad (1)$$

2^0. Tenglikda *yo'naltirilgan kesmaning* yoki *boshi* $A(x_1;y_1)$ va *oxiri* $B(x_2;y_2)$ bo'lgan $\overline{AB}$ *vektorning* koordinata o'qlaridagi *proyektsiyalari:*

$$\text{Pr}_x\overline{AB}=X=x_2-x_1, \qquad \text{Pr}_y\overline{AB}=Y=y_2-y_1 \qquad (2)$$

3^0. *Kesmani berilgan nisbatta bo'lish:* $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalar berilgan AB kesmani $AN:NB=\lambda$ nisbatda bo'luvchi $N(x;y)$ nuqtaning koordinatalari ushbu:

$$x=\frac{x_1+\lambda x_2}{1+\lambda}, \qquad y=\frac{y_1+\lambda y_1}{1+\lambda} \qquad (3)$$

formulalar bilan aniqlanadi. Xususiy holda kesmani teng ikkiga, ya'ni $\lambda=1:1=1$ nisbatda bo'lganda

$$\text{x}=\frac{x_1+x_2}{2}, \quad \text{y}=\frac{y_1+y_2}{2} \qquad (4)$$

4^0. Uchlari $A(x_1;\, y_1)$, $B(x_2;\, y_2)$, $C(x_3;\, y_3)$, ..., $F(x_n;\, y_n)$ nuqtalarda bo'lgan *ko'pburchak yuzi:*

$$S=\pm\frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}+\begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix}+\ldots+\begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix}\right] \qquad (5)$$

ga teng.

5^0. *To'g'ri chiziqning burchak koeffitsientli tenglamasi:*

$$y=kx+b \qquad (6)$$

k parametr to'g'ri chiziqning Ox o'qqa og'ish burchagi α ning tangensiga teng bo`lib ($k=tg\alpha$), to'g'ri chiziqning burchak koeffitsenti, ba'zan qiyaligi deyiladi. b parametr boshlang'ich ordinata yoki Oy o'q ajratgan kesma kattaligi .

6^0. *To'g'ri chiziqning umumiy tenglamasi:*

$Ax+Bx+C=0 \qquad (A^2+B^2 \neq 0)$ (7)

Xususiy hollar:

a) $C=0$ bo'lsa, $y=-\frac{A}{B}x$ to'g'ri chiziq koordinatalar boshidan o'tadi;

b) $B=0$ bo'lsa, $x=-\frac{C}{A}=a$ to'g'ri chiziq 0x o'qqa parallel bo'ladi;

c) $A=0$ bo'lsa, $y=-\frac{C}{B}=b$ to'g'ri chiziq $0y$ o'qqa parallel bo'ladi;

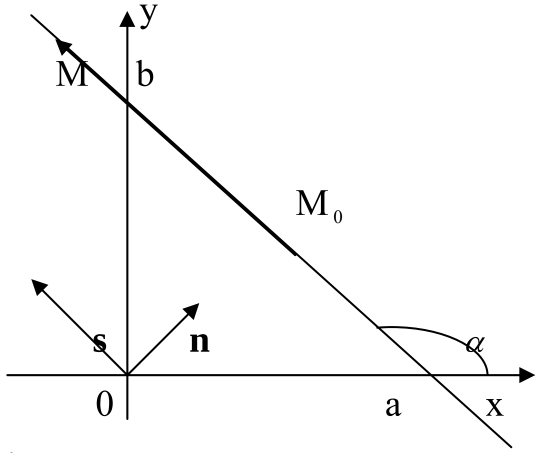
d) $B=C=0$ bo'lsa, $Ax=0$ yoki $x=0$ - to'g'ri chiziq $0y$ o'qdan iborat;

e) $A=C=0$ bo'lsa, $By=0$ yoki $y=0$ - to'g'ri chiziq $0x$ o'qdan o'tadi.

7^0. *To'g'ri chiziqning o'qlardan ajratgan kesmalari bo'yicha tenlamasi*:

$$\frac{x}{a}+\frac{y}{b}=1 \qquad (8)$$

Bu yerda *a* va *b* - to'g'ri chiziqning o'qlardan kesgan kesmalarining kattaliklari.



8^0. *To'g'ri chiziqning vektor parametrli tenglamasi:*

$$\overline{M_0M}=t\boldsymbol{s} \qquad (9)$$

Bu yerda *M(x;y)* to'g'ri chiziqning ixtiyoriy nuqtasi $\overline{M_0M}$ $(x-x_0;\ y-y_0)$ vektor va ***s****(m;n)* yo'naltiruvchi vektori o'zaro kollinear, t-ixtiyoriy haqiqiy son yoki parametr.

9^0. (9) tenglamani koordinatalarda

$$\begin{cases} x-x_0=tm \\ y-y_0=tn \end{cases} \qquad (10)$$

ifodalab, *to'g'ri chiziqning parametrli tenglamasini* hosil qilish mumkin.

10^0. (10) tenglamalarda t parametr yo'qotilsa, to'g'ri chiziqning kanonik teglamasi hosil bo'ladi:

$$\frac{x-x_0}{m}=\frac{y-y_0}{n} \qquad (11)$$

11^0. Agar $|\bar{a}|=P$ $(P\geq 0)$, $\quad v=\frac{\bar{a}}{P}=(\text{Cos}\,\alpha,\ \text{Cos}\,\beta)$ $\quad \bar{a}$ normal radius vektorninig birlik vektori bo'lib, to'g'ri chiziqning ixtiyoriy *M(x;y)* nuqtasining mos radius vektori $\bar{r}$(x;y) bo'lsa, u holda $\bar{r}$ radius vektorning $\bar{a}$ yoki $\bar{v}$ vektordagi sonli proyektsiyasi *P* ga teng:

$$\text{P}r_v\bar{r}=P, \quad \text{yoki} \quad |v|\text{P}r_v\bar{r}=P, \quad \text{yoki} \quad (r\cdot v)=P\ (P\geq 0) \qquad (12)$$

Bu tenglama to'g'ri chiziqning *vektor ko'rinishdagi tenglamasi* deyiladi.

(12) tenglama koordinatalarda

$$xCos\,\alpha+yCos\,\beta=P \quad \text{yoki} \quad xCos\,\alpha+ySin\,\alpha=P\ (P\geq 0) \qquad (13)$$

ko'rinishni oladi. Bunda α - $\bar{a}$ yoki $\bar{v}$ vektorning *Ox* o'qining musbat yo'nalishi bilan hosil qilgan burchak kattaligi. (13) shakldagi tenglama to'g'ri chiziqning *normal tenglamasi* deyiladi.

12^0. (7) shakldagi tenglamadan (13) shakldagi tenglamaga o'tish uchun umumiy ko'rinishdagi tenglama normallovchi ko'paytuvchi deb ataladigan $\mu=\pm\frac{1}{\sqrt{A^2+B^2}}$ songa ko'paytiriladi, bunda "+" yoki "–" ishoradan *C* ozod had ishorasining qarama–qarshisi tanlanadi, aks holda *P*= - μ C ≥ 0 munosabat bajarilmaydi.

Masala: *3x+4y-8=0* tenglamani normal ko'rinishga keltiring .

Berilgan umumiy shakldagi tenglama uchun normallovchi ko'paytuvchi

$$\mu=\pm\frac{1}{\sqrt{3^2+4^2}}=\frac{1}{5}.$$

Tenglamani, $\mu=\frac{1}{5}$ ga ko'paytiramiz, natijada to'g'ri chiziq tenglamasi quyidagi ko'rinishda normal holga keltiriladi:

$$\frac{3}{5}x+\frac{4}{5}y=\frac{8}{5}.$$

13^0. $y=k_1x+b_1$ to'g'ri chiziqdan $y=k_2x+b_2$ to'g'ri chiziqqacha soat strelkasiga qarshi yo'nalishda hisoblanuvchi φ *burchak*

$$tg\varphi=\frac{k_2-k_1}{1+k_1\,k_2} \tag{14}$$

formula bilan aniqlanadi.

14^0. $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ tenglamalar bilan berilgan to'g'ri chiziqlar uchun (14) formula quyidagi ko'rinishga ega bo'ladi:

$$tg\varphi=\frac{A_1B_2-A_2B_1}{A_1A_2+B_1B_2} \tag{15}$$

yoki $$Cos\varphi=\frac{(n_1\cdot n_2)}{|n_1|\cdot|n_2|}=\frac{A_1\,A_2+B_1B_2}{\sqrt{A^2{}_1+B_1{}^2}\bullet\sqrt{A^2+B^2}} \tag{16}$$

15^0. To'g'ri chiziqning *parallellik* sharti:

$$k_1=k_2 \quad \text{yoki} \quad \frac{A_1}{A_2}=\frac{B_1}{B_2} \tag{17}$$

16^0. To'g'ri chiziqning *perpendikulyarlik* sharti:

$$k_1\bullet k_2=-1 \quad \text{yoki} \quad A_1A_2+B_1B_2=0 \tag{18}$$

17^0. Berilgan $A(x_1;y_1)$ nuqtadan o'tuvchi *to'g'ri chiziqlar dastasining tenglamasi:* $y-y_1=k(x-x_1)$ (19)

18^0. Berilgan ikki $A(x_1;y_1)$ va $B(x_2;y_2)$ *nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi:* $$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1} \tag{20}$$

19^0. Parallel bo'lmagan ikki $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ to'g'ri chiziqning *kesishish nuqtasini* topish uchun ularning tenglamalarini birgalikda yechish bilan

$$x=\frac{\begin{vmatrix}-C_1 & B_1\\ -C_2 & B_2\end{vmatrix}}{\begin{vmatrix}A_1 & B_1\\ A_2 & B_2\end{vmatrix}},\quad y=\frac{\begin{vmatrix}A_1 & -C_1\\ A_2 & -C_2\end{vmatrix}}{\begin{vmatrix}A_1 & B_1\\ A_2 & B_2\end{vmatrix}} \tag{21}$$

ni hosil qilamiz.

20^0. *$(x_0;y_0)$ nuqtadan to'g'ri chiziqqacha bo'lgan d masofani topish uchun* to'g'ri chiziq normal tenglamasining chap tomonidagi o'zgaruvchi koordinatalar o'rniga *$(x_0;y_0)$* koordinatalarni qo'yib, hosil bo'lgan sonning absolyut qiymatini olamiz, ya'ni

$$d=|x_0\cos\beta + y_0\sin\beta - P| \qquad (22)$$

yoki
$$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}} \qquad (23)$$

21^0. $Ax+By+C=0$ va $A_1x+B_1y+C_1=0$ to'g'ri chiziqlar orasidagi burchaklar *bissektrissalarining* tenglamalari:

$$\frac{Ax+By+C}{\sqrt{A^2+B^2}}=\pm\frac{A_1x+B_1y+C_1}{\sqrt{A_1^{\,2}+B_1^{\,2}}} \qquad (24)$$

22^0. Berilgan ikki to'g'ri chiziqning kesishish nuqtasidan o'tuvchi to'g'ri chiziqlar *dastasining tenglamasi:*

$$\alpha(Ax+By+C)+\beta(A_1x+B_1y+C_1)=0 \qquad (25)$$

$\alpha=1$ deb olish mumkin, u holda biz (25) dastadan berilgan to'g'ri chiziqlardan ikkinchisini yo'qatgan bo'lamiz, ya'ni u vaqtda (25) dan ikkinchi to'g'ri chiziqning tenglamasini hosil qila olmaymiz.

Mustaqil yechish uchun masalalar:

1. $\frac{x+2\sqrt{5}}{4}+\frac{y-2\sqrt{5}}{2}=0$ to'g'ri chiziq berilgan. To`g`ri chiziqning

   a) umumiy tenglamasi,

   b) burchak koeffitsientli tenglamasi,

   c) kesmalarga nisbatan tenglamasini yozing.

2. $4x+3y-36=0$ to'g'ri chiziq, koordinata o'qlari bilan hosil qilgan uchburchakning yuzini toping.

3. To'g'ri chiziq koordinata o'qlaridan teng kesmalar ajratadi. Agar to'g'ri chiziq koordinata o'qlari bilan hosil qilgan uchburchak yuzi 8 *kv.birl.* bo'lsa, to'g'ri chiziq tenglamasini yozing.

4. $A(2;5)$ nuqtadan o’tuvchi va ordinata o’qida $b=7$ kesma ajratuvchi to’g’ri chiziq tenglamasini yozing.

5. Agar to’g’ri chiziq koordinata o’qlaridan teng kesmalar ajratsa va to’g’ri chiziqni koordinata o’qlari orasidagi kesmasi $5\sqrt{2}$ ga teng bo’lsa, to’g’ri chiziq tenglamasini yozing.

6. $y=-2$, $y=4$ to’g’ri chiziqlar $3x-4y-5=0$ to’g’ri chiziqni A va B nuqtalarda kesib o’tadi. $\overline{AB}$ vektorni uzunligi va uni koordinata o’qlaridagi proyektsiyalarini toping.

7. To’g’i chiziqlar orasidagi burchakni toping:

1) $\begin{cases} y = 2x - 3 \\ y = \dfrac{1}{2}x + 1 \end{cases}$ 2) $\begin{cases} 5x - \ y + 7 = 0 \\ 2x - 3y + 1 = 0 \end{cases}$ 3) $\begin{cases} 2x + y = 0 \\ y = 3x - 4 \end{cases}$

8. $3x-2y+7=0,$ $6x-4y-9=0,$ $6x+4y-5=0,$ $2x+3y-6=0$ to’g’ri chiziqlar orasidan parallel va perpendikulyar to’g’ri chiziqlarni aniqlang.

9. $A(2;3)$ nuqtadan o’tuvchi to’g’ri chiziqlar dastasini yozing. Bu dastadan Ox o’qi bilan 1) 45^0, 2) 60^0, 3) 135^0, 4) 0^0 burchaklar tashkil etuvchi to’g’ri chiziqni toping.

10. $A(-2;5)$ nuqta va $2x-y=0$ to’g’ri chiziqni yasang. A nuqtadan o’tuvchi va

1) berilgan to’g’ri chiziqqa parallel

2) berilgan to’g’ri chiziqqa perpendikulyar to’g’ri chiziq tenglamasini yozing.

11. $2x-5y-10=0$ to’g’ri chiziqni koordinata o’qlari bilan kesishish nuqtalariga perpendikulyar qo’yilgan. Ularning tenglamasini yozing.

12. $A(-1;3)$ va $B(4;-2)$ nuqtalardan o’tuvchi to’g’ri chiziq tenglamasini yozing.

13. Uchlari $A(-2;0)$, $B(4;-2)$ va $C(4;2)$ bo’lgan uchburchakka BD balandlik va BE mediana o’tkazilgan. AC tomon, BE mediana va BD balandlik tenglamalarini yozing.

14. Uchburchak tomonlari quyidagi tenglamalar bilan berilgan:

$x+3y=0$, $x=3$, $x-2y+3=0$. Uchburchakni burchaklari va uchlarini toping.

15. Kvadrat tomonlaridan birining tenglamasi $x+3y-7=0$ va diogonallari kesishgan nuqta $P(0;-1)$ berilgan. Kvadratning qolgan uchta tomon tenglamalarini yozing.

16. Romb tomonlaridan birining tenglamasi $5x+2y-9=0$. Agar romb diogonallari $O(0;0)$ da kesishgan bo'lib, ulardan birining tenglamasi $y=2x$ bo'lsa, pombning qolgan uchta tomon tenglamasini yozing.

17. Uchburchak tomonlarining o'rtasi berilgan $P(1;2)$ - AB tomonining o'rtasi, $R(-4;3)$ - BC tomonining o'rtasi, $Q(5;-1)$ - AC tomonining o'rtasi, CF balandlik va AR mediana kesishgan nuqta topilsin.

18. Rombning ikki qarama-qarshi uchlarining koordinatalari berilgan, $A(1;-4)$ $C(-1;3)$. Romb diogonallarining tenglamasini yozing.

19. Agar $A(-5;5)$ va $B(3;1)$ uchburchakning uchlari, $D(2;5)$ esa balandliklari kesishgan nuqta bo'lsa, uchburchak tomonlarining tenglamasini yozing.

20. $2x+2y-5=0$ to'g'ri chiziq Ox o'qining musbat yo'nalishi bilan qanday burchak hosil qiladi?

21. Oy o'qidan $b=1$ birlikka teng kesma ajratuvchi Ox o'qining musbat yo'nalishi bilan $\alpha=\frac{2\pi}{3}$ burchak hosil qiluvchi to'g'ri chiziq tenglamasini yozing.

22. Koordinata boshidan va $A(-2;-3)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini yozing.

23. $M(-3;-4)$ nuqtadan o'tuvchi koordinata o'qlariga parallel to'g'ri chiziqlar tenglamasini yozing.

24. $O(0;0)$ va $A(-3;0)$ nuqtalar berilgan OA kesmada parallelogramm yasalgan, uning diogonallari $B(0;2)$ nuqtada kesishadi. Parallelogramm tomonlari va diogonallari tenglamasini yozing.

25. Tomonlari 8 *sm* va 2 *sm* bo'lgan teng yonli trapetsiyaning o'tkir burchagi 45^0. Trapetsiyaning katta asosi *Ox* o'qida yotsa, *Oy* o'qi esa trapetsiyaning simmetriya o'qi bo'lsa, trapetsiyaning tomonlari tenglamasini yozing.

26. Agar to'g'ri chiziq koordinata o'qlari bilan hosil qilgan uchburchak yuzi 6 *kv.b.* bo'lsa va to'g'ri chiziq (-4;6) nuqtadan o'tsa, uning tenglamasini yozing.

27. To'g'ri chiziqlar orasidagi burchakni toping:

a) $\begin{cases} 3x+2y=0 \\ 6x+4y+9=0 \end{cases}$ b) $\begin{cases} 3x-4y=0 \\ 8x+6y=11 \end{cases}$

28. Uchlari *A*(-2;0), *B*(2;4) va *C*(4;0) bo'lgan uchburchak berilgan. Uchburchak tomonlari, *AE* medianasi, *BD* balandlik tenglamalarini, *AE* mediana uzunligini toping.

29. Tomonlari *x+y=4,* *3x-y=0,* *x-3y-8=0* tenglamalar bilan berilgan uchburchakni burchaklari, uchlari va uchburchakni yuzini toping.

30. Koordinatalar boshidan *2x+y=a* to'g'ri chiziq bilan teng yonli uchburchak hosil qiluvchi ikki o'zaro perpendikulyar to'g'ri chiziq o'tkazilgan. Shu uchburchakning yuzini toping.

Ko'rsatma: 2x+y=3 bilan y=kx va $y=-\frac{x}{k}$ *to'g'ri chiziqlarning kesishgan nuqtalari M va N ning koordinatalarini topgandan so'ng OM=ON tenglikdan k ni topish kerak.*

31. Uchburchak *AB* tomonining tenglamasi *x-3y+3=0* va *AC* tomonining tenglamasi *x+3y+3=0* hamda *AD* balandligining asosi *D*(-1;3) berilgan bo'lsa, uchburchakning ikki burchaklari topilsin.

32. Romb ikki tomonining tenglamalari *x+2y=4* va *x+2y=10* hamda diogonallaridan birining tenglamasi *y=x+2* ma'lum bo'lsa, romb uchlarining koordinatalari hisoblansin.

Javoblar:

1. a) $x+2y-2\sqrt{5}=0$ b) $-\frac{1}{2}x+\sqrt{5}$ c) $\frac{x}{2\sqrt{5}}+\frac{y}{\sqrt{5}}=1$

2. 54 *km. birlik.*

3. $x+y-4=0$

4. $x+y-7=0$

5. $x+y-5=0$ $x+y+5=0$

6. $|AB|=10$ $\Pr_{ox} AB=8$ $\Pr_{oy} AB=6$

7. 1) $arctg\frac{3}{4}$ 2) 45^0 3) 45^0

10. $y=2x+9$ $y=-\frac{1}{2}x+4$

11. $5x+2y+(-4)=0$ $5x+2y=25$

12. $x+y-2=0$

13. AC: $x-3y+2=0;$ BD: $3x+y=12;$ BE: $x+y-2=0$

14. (3;-1), (3;3), $\left(-\frac{9}{5};\frac{3}{5}\right)$, 45^0, $71^0 31'$, $63^0 24'$

20. 135^0

21. $\sqrt{3x}+y-1=0$

22. $3x+2y=0$

23. $x+3=0,$ $y+4=0$

24. $y=0,$ $4x-3y=0,$ $4x-3y+12=0,$ $y=4$

25. $x+y-4=0,$ $x-y+4=0,$ $y=3,$ $y=0$

26. $\frac{x}{4}+\frac{y}{3}=1,$ $\frac{x}{-2}+\frac{y}{-6}=1$

27. a) 0^0, b) 90^0

28. AE: $2x-5y=-4,$ AD: $x-2y=-2$ $|\overline{AE}|=\sqrt{29}$

29. $tgA=\frac{4}{3}$ $tgB=tgC=2.$ $S=16$ *kv.birlik*

30. $\frac{a^2}{5}$ *kv.birlik*

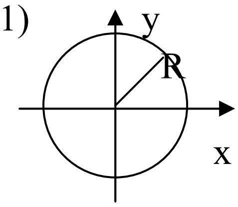
31. $A=36^0 52'$ $B=128^0 52'$

32. (0; 2), (4; 0) (2; 4), (-2; 6).

13. IKKINCHI TARTIBLI EGRI CHIZIQLAR

1^0. Markazi koordinata boshida, radiusi *R* bo'lgan *aylana* tenglamasi(1-rasm):

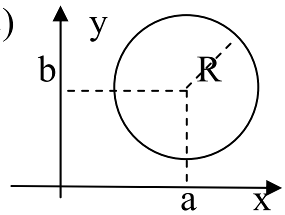
$$x^2 + y^2 = R^2 \qquad (1)$$



1- rasm.

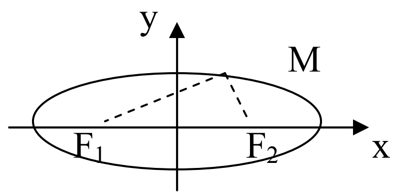
2^0. Markazi *(a;b)* nuqtada, radiusi *R* bo'lgan *aylana* tenglamasi (2-rasm):

$$(x-a)^2 + (y-b)^2 = R^2 \qquad (2)$$



2-rasm.

3^0. ***Ellips*** (3-rasm):



3-rasm.

Fokus deb ataluvchi $F_1(-c;0)$ va $F_2(c;0)$ nuqtalardan $|F_1M|+|F_2M|=2a$ masofaga teng ixtiyoriy *M(x;y)* nuqtalar to'plami ellips deyeladi. F_1M va F_2M kesmalar *fokal radiuslar* deyiladi, hamda

$$|F_1M| = \sqrt{(x+c)^2 + y^2}$$

$$|F_2M| = \sqrt{(x-c)^2 + y^2} \qquad (3)$$

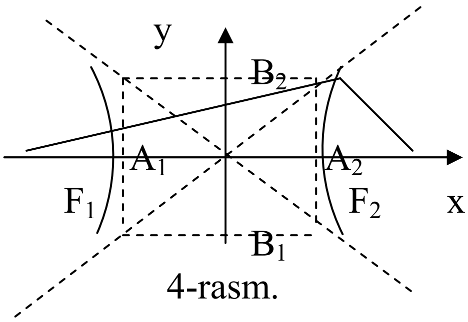
ga teng. Ellipsning *kanonik tenglamasi*:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (4)$$

bunda $b = \sqrt{a^2 - c^2}$. Ellipsning *kichik yarim o'qi a*, *katta yarim o'qi b*. *Markazi* esa *O(0;0)* – koordinata boshi. Ellipsning *uchlari (-a;0), (a;0), (0;-b), (0;b)*.

Ellipsning *simmetriya markazi O(0;0)*, *simmetriya o'qlari Ox, Oy* o'qlar. Ellipsning *ekstsentrisiteti* $\varepsilon = \frac{a}{c} < 1$ ga aytiladi.

4^0. ***Giperbola*** (4-rasm):



4-rasm.

Fokuslar F_1(-c;0) va *F_2(c;0)* gacha bo'lgan masofalar ayirmasi.

$$\left\| F_1M \right| - \left| F_2M \right\| = 2a$$

ga teng ixtiyoriy *M(x;y)* nuqtalar to'plamiga *giperbola* deyiladi.

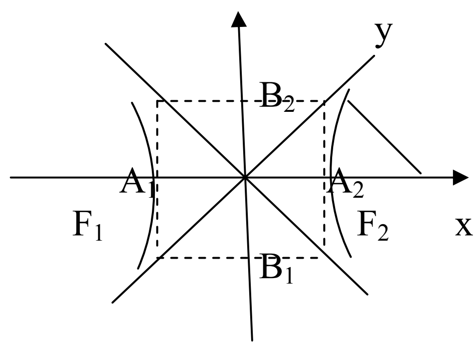
Kanonik tenglamasi:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad (5)$$

bunda $b = \sqrt{c^2 - a^2}$. *Haqiqiy uchlari*: *A_1(-a;0), A_2(a;0); mavhum uchlari:B_1(0;-b), B_2(0;b)*. Giperbolaning asimtotalari: $y = \frac{b}{a}x$ (I va III choraklardan o'tadi) va $y = -\frac{b}{a}x$ (II va IV choraklardan o'tadi).

Yarim o'qlari teng, ya'ni $a = b$ giperbolaga *teng tomonli giperbola* deyiladi (5-rasm) va $x^2 - y^2 = a^2$ ko'rinishida ifodalanadi.

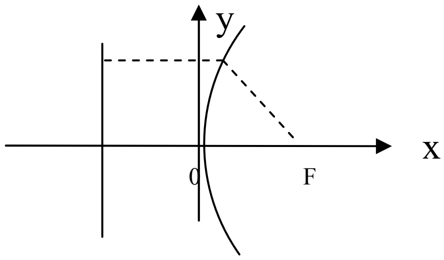
Ekstrisentrisiteti: $\varepsilon = \frac{c}{a} > 1$



5-rasm.

5^0. ***Parabola*** (6-rasm): *Fokusi* $F(\frac{p}{2};0)$ dan va *direktrisasi* $x=-\frac{p}{2}$ to'g'ri chizig'igacha teng masofada yotuvchi ixtiyoriy *M(x;y)* nuqtalar to'plamiga *parabola* deyiladi. Parabolaning *kanonik tenglamasi:*

$$y^2 = 2px \qquad (6)$$



6-rasm.

Parabolaning uchi koordinata boshi *O(0;0)*. *Fokusdan direktrisa* to'g'ri chizig'igacha bo'lgan masofa *p* ga teng.

Mustaqil yechish uchun masalalar:

1. *A(-4;6)* nuqta berilgan. Diametri *OA* kesma bo'lgan aylana tenglamasini tuzing.
2. *A(-6;0)* nuqtadan o'tuvchi va *Oy* o'qiga koordinatalar boshida urinuvchi aylana tenglamasini tuzing.
3. $x^2+y^2+4x-6y=0$ aylananing *Oy* o'qi bilan kesishgan nuqtalariga o'tkazilgan radiuslari orasidagi burchak topilsin.
4. *A (-1;3), B (0;2)* va *C (1;-1)* nuqtalardan o'tuvchi aylana tenglamasi yozilsin. *Ko`rsatma*: Izlanayotgan aylananing tenglamasini $x^2+y^2+mx+ny+p=0$ ko`rinishida yozib, undagi *x* va *y* lar o`rniga berilgan har bir nuqtaning koordinatalarini qo`ygandan so`ng *m, n* va *p* larni topish kerak.
5. *A(4;4)* nuqtadan va $x^2+y^2+4x-4y=0$ aylana bilan *y=-x* to'g'ri chiziqning kesishgan nuqtalaridan o'tuvchi aylana tenglamasi yozilsin.

Ellips.

6. Katta o'qi 8 va kichik o'qi 6 bo'lgan ellipsning tenglamasini tuzing. Ellips tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ dan masalani shartiga ko'ra topamiz. *2a=8, 2b=6;* ya'ni *a=4, b=3*. Bularni ellips tenglamasiga qo'yamiz. $\frac{x^2}{16}+\frac{y^2}{9}=1$

7. $4x^2+9y^2=36$ ellips tenglamasidan uning o'qlari, fokuslari va ekstsentrisitetini toping.

$4x^2+9y^2=36$

Tenglamani ikkala tomonini 36 ga bo`lamiz. $\frac{x^2}{9}+\frac{y^2}{4}=1$ $a^2=9;$ $a=+3$;

$b^2=4;$ $b=+2;$ $c^2=a^2-b^2$ dan $c^2=9-4=5;$ $c=+\sqrt{5};$ $\varepsilon=\frac{\sqrt{5}}{3}$.

Demak, *2a=6; 2b=4;*

$F_1(\sqrt{5},0);$ $F_2(-\sqrt{5},0);$ $\varepsilon=\frac{\sqrt{5}}{3}\prec 1$

8. Katta yarim o'qi *a=5* va *C* parametri

1) 4.8; 2) 4; 3) 3; 4)1.4; 5) 0

Berilgan ellipsni kanonik tenglamasini yozing. Har bir ellipsni chizing va ularning ekstsentrisitetini toping.

9. Yer fokuslaridan birida Quyosh joylashgan ellips bo`yicha harakat qiladi. Quyoshdan Yergacha bo'lgan eng kichik masofa taxminan 147.5 *million km* ga, eng katta masofa 152.5 *million km* ga teng bo'lsa, Yer orbitasining katta yarim o'qi va ekstsentrisiteti topilsin.

10. Ekstsentrisiteti $\varepsilon=\frac{3}{4}$ bo'lgan va $M=(-4;\sqrt{21})$ nuqtadan o'tuvchi ellips tenglamasini yozing va *M* nuqtaning fokal radius-vektorlarini toping.

11. Koordinata o'qlariga nisbatan simmetrik bo'lgan ellips $M(2;\sqrt{3})$ va $B(0;2)$ nuqtalaridan o'tadi. Uning tenglamasi yozilsin va *M* nuqtadan fokuslarigacha bo'lagan masofa topilsin.

12. $9x^2+25y^2=225$ ellipsda shunday $M(x;y)$ nuqta topilsinki, undan o'ng fokusgacha bo'lgan masofa chap fokusgacha bo'lgan masofadan 4 marta katta bo'lsin.

13. Agar ellipsning fokuslari orasidagi masofa uning katta va kichik yarim o’qlarining uchlari orasidagi masofaga teng bo’lsa, uning ekstsentrisiteti topilsin.

Giperbola.

14. Fokuslari orasidagi masofa $2\sqrt{11}$ bo’lib, o’zi (9;-4) nuqtadan o’tgan giperbola tenglamasini tuzing.

Shartga asosan $2c=2\sqrt{11}$, bundan $c=\sqrt{11}$. Giperbola (9;-4) nuqtadan o`tganligi uchun bu nuqta giperbola tenglamasini qanoatlantiradi, ya’ni

$$\frac{9^2}{a^2}-\frac{(-4)^2}{b^2}=1$$

$81b^2-16a^2=a^2b^2$

$a^2+b^2=c^2=11$ buni ellips tenglamasiga qo`yamiz.

$81b^2-16(11-b^2)=(11-b^2)b^2$

$b^4-86b^2-176=0$

$b_1^2=2;\ \ b_2^2=-83$

$a^2=11-b^2=9$

Demak, giperbola tenglamasi quyidagicha bo`ladi: $\frac{x^2}{9}-\frac{y^2}{2}=1$

15. $16x^2-2y^2=400$ giperbola tenglamasi berilgan. Uning o’qlari, fokuslari, ekstsentrisitetini toping va asimptotasinining tenglamasini tuzing.

16. Giperbolaning ekstsentrisiteti $\sqrt{2}$ ga teng va $M\ (2a;\ a\sqrt{3})$ nuqtadan o’tadi. Giperbolani sodda teglamasini tuzing.

17. Giperbolani fokuslari $F_1(-\sqrt{7};0)$ va $F_2\ (\sqrt{7};0)$ nuqtalarda joylashgan. Agar Giperbola $A(2;0)$ nuqtadan o’tsa, uning asimptotalari tenglamasini tuzing.

18. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning fokusidan asimptotalarigacha bo’lgan masofalar va asimptotalari orasidagi burchak topilsin.

19. Biror uchidan fokuslarigacha bo’lgan masofalari 9 va 1 ga teng bo’lgan giperbolaning kanonik tenglamasi yozilsin.

20. $M\left(6;\frac{3}{2}\sqrt{5}\right)$ nuqtadan o'tuvchi, koordinata o'qlariga nisbatan simmetrik bo'lgan giperbolaning haqiqiy yarim o'qi $a=4$. Giperbolaning chap fokusidan asimptotalariga tushirilgan perpendikulyarning tenglamalari yozilsin.

Parabola.

21. Parabola (3;5) nuqtadan o'tadi. Uning kanonik tenglamasini yozing.

Parabola (3;5) nuqtadan o'tganligi uchun tenglamasini qanoatlantiradi.

$y^2 = 2px$ $\quad x=3 \quad y=5$

$25 = 2\cdot p\cdot 3 \quad 25=6p \quad p=\frac{25}{6}$

Demak, $y^2 = 2\cdot\frac{25}{6}x \qquad y^2=\frac{25}{3}x$ - parabolaning kanonik tenglamasi.

22. 1) (0;0) va (1;-3) nuqtalardan o'tuvchi va Ox o'qqa nisbatan simmetrik;

2) (0;0) va (2;-4) nuqtalardan o'tuvchi va Oy o'qqa nisbatan simmetrik bo'lgan parabola tenglamasi yozilsin.

23. Agar parabola $x=y$ to'g'ri chiziq va $x^2+6x+y^2=0$ aylananing kesishish nuqtalaridan o'tsa, uning tenglamasi va direktrisasini yozing.

24. $y^2=6x$ parabolada fokal radius vektor 4.5 ga teng bo'lgan nuqtani toping.

25. $A(-1;3)$, $B(0;2)$ va $C(1;-1)$ nuqtalardan o'tuvchi aylana tenglamasini yozing.

26. Ellips $M(2\sqrt{3};\sqrt{6})$ va $A(6;0)$ nutalardan o'tadi. Uning tenglamasini, ekstsentrisiteti va M nuqtadan fokuslargacha bo'lgan masofani yozing.

27. $x^2+4y^2=4$ ellipsning, markazi shu ellipsning "yuqori" uchida bo'lgan va uning fokuslaridan o'tuvchi aylana bilan umumiy nuqtalari topilsin.

28. $y^2=a^2+x^2$ giperbola fokuslari koordinatalarini va asimptotalari orasidagi burchakni toping.

29. Uchlari $\frac{x^2}{25}+\frac{y^2}{9}=1$ ellipsning fokuslarida, fokuslari esa uning uchlarida bo'lgan giperbola tenglamasini yozing.

30. 1) (0;0) va (-1;2) nuqtalardan o'tuvchi va *Ox* o'qiga simmetrik.

2) (0;0) va (2;4) nuqtalardan o'tuvchi va *Oy* o'qiga simmetrik bo'lgan parabola tenglamasini yozing.

31. Markazi $y^2 = 2px$ parabolaning fokusida bo'lib, parabola direktrisasiga urinuvchi aylana tenglamasi yozilsin. Parabola va aylananing kesishgan nuqtalari topilsin.

Javoblar:

1. $x^2+y^2+4x-6y=0$

2. $x^2+y^2+6x=0$

3. tgα = -2,4; $\alpha = 112^0 37$`

4. $(x+4)^2+(y+1)^2=25$

5. $x^2+y^2-8y=0$

8. *b*=1,4; 3; 4; 4,8; 5 $\qquad \varepsilon$=0,96; 0,8; 0,6; 0,28; 0

9. *a*=150 *mln. kv.* $\varepsilon = \frac{1}{60}$

10. $\frac{x^2}{64}+\frac{y^2}{28}=1$; $r_1=11$; $r_2=5$

11. $\frac{x^2}{16}+\frac{y^2}{4}=1$; $\varepsilon=\frac{\sqrt{3}}{2}$; $r_1=4-\sqrt{3}$; $r_2=4+\sqrt{3}$;

12. $\left(-\frac{15}{4};\frac{\sqrt{63}}{4}\right)$

13. $\sqrt{0,4}$

15. $x^2-y^2=4$

16. $x^2-y^2=a^2$

18. b; $2arctg\frac{b}{a}$

19. $\frac{x^2}{16}-\frac{y^2}{9}=1$ $\left(yoki\ \frac{x^2}{9}-\frac{y^2}{16}=-1\right)$

20. $y=\pm\frac{4}{3}(x+5)$

22. 1) $y^2=9x$; 2) $y=-x^2$

23. $y^2=-3x$

24. $\left(3;\pm3\sqrt{2}\right)$ *Ko`rsatma: izlanayotgan aylananing tenglamasini $x^2+y^2+mx+ny+p=0$ ko`rinishda yozib olish kerak.*

25. $(x+4)^2+(y+1)^2=25$

26. $\frac{x^2}{36}+\frac{y^2}{9}=1$; $\varepsilon=\frac{\sqrt{3}}{2}$; $r_1=3$; $r_2=9$;

27. $\left(\pm\frac{4\sqrt{2}}{3};\frac{1}{3}\right)$

28. $\left(0;\pm a\sqrt{2}\right)$; 90^0

29. $\frac{x^2}{16}-\frac{y^2}{9}=1$

30. 1) $y^2=-4x$; 2) $y=x^2$

31. $\left(x-\frac{p}{2}\right)^2+y^2=p^2$; $\left(\frac{p}{2};\pm p\right)$

14. FAZODA TEKISLIK TENGLAMALARI

1^0. Uch o'lchovli *Oxyz* koordinatalar sistemasida berilgan tekislik tenglamasi:

$Ax+By+Cz+D=0 \quad (A^2+B^2+C^2\neq 0)$ (1)

$\overline{N}(A;B;C)$ tekislikka perpendikulyar bo'lgan *normal vektor* deyiladi.

2^0. $M_1(x_1;y_1;z_1)$ *nuqtadan o'tuvchi* va $\overline{N}(A;B;C)$ vektorga perpendikulyar *tekislik tenglamasi*:

$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$ (2)

3^0. *Ax+By+Cz+D=0* tenglamaning maxsus hollari:

1) *D*=0 bo'lganda, *Ax+By+Cz=0* tekislik koordinatalar boshidan o`tadi;

2) *C*=0 bo'lganda, *Ax+By+=D=0* tekislik *Oz* o'qiga parallel;

3) *C*=*D*=0 bo'lganda, *Ax+By=0* tekislik *Oz* o'qidan o'tadi;

4) *B*=*C*=0 bo'lganda, *Ax+D*=0 tekislik *yOz* tekislikka parallel;

5) Koordinata tekisliklarining tenglamalari: *x*=0, *y*=0 va *z*=0.

4^0. *Tekislikning koordinata o'qlaridan ajratgan kesmalar bo'yicha tenglamasi*:

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \quad (3)$$

5^0. *Ikki tekislik orasidagi burchak*:

$$Cos\alpha=\pm\frac{AA_1+BB_1+CC_1}{\sqrt{A^2+B^2+C^2}\bullet\sqrt{A_1^2+B_1^2+C_1^2}}=\pm\frac{(\overline{N}\bullet\overline{N}_1)}{|\overline{N}|\bullet|\overline{N}_1|} \quad (4)$$

formuladan topiladi, bunda $\overline{N}$ va $\overline{N}_1$ mos ravishda *Ax+By+Cz+D=0* va $A_1x+B_1y+C_1z+D_1=0$ tekisliklarga normal vektorlar.

Parallellik sharti: $\dfrac{A}{A_1}=\dfrac{B}{B_1}=\dfrac{C}{C_1}$ (5)

Perpendikulyarlik sharti: $AA_1+BB_1+CC_1=0$ (6)

6^0. $M_0(x_0;y_0;z_0)$ *nuqtadan* o'tuvchi *Ax+By+Cz+D=0* *tekislikkacha* bo'lgan *masofa*: $d=\dfrac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$ (7)

7^0. Berilgan ikki tekislikning *kesishgan chizig'idan* o'tuvchi barcha *tekisliklar dastasining tenglamasi* quyidagicha yoziladi:

$$\alpha(Ax+By+Cz+D)+\beta(A_1x+B_1y+C_1z+D)=0 \qquad (8)$$

8^0. Bir to'g'ri chiziqda yotmaydigan *uchta* $(x_1;y_1;z_1)$, $(x_2;y_2;z_2)$ va $(x_3;y_3;z_3)$ *nuqtadan o'tuvchi tekislik tenglamalari*:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0 \qquad (9)$$

Mustaqil yechish uchun masalalar:

1. $M_1(0;-1;3)$ va $M_2(1;3;5)$ nuqtalar berilgan, M_1 nuqtadan o'tuvchi va $\boldsymbol{N}=\overline{M_1M_2}$ vektorga perpendikulyar tekislik tenglamasi yozilsin.
2. $M(a;a;0)$ nuqtadan o'tuvchi va $\overline{OM}$ vektorga perpendikulyar tekislik tenglamasi yozilsin.
3. $A(a;-\frac{a}{2};a)$ va $B(0;\frac{a}{2};0)$ nuqtadan teng uzoqlikda bo'lgan nuqtalar geometrik o'rnining tenglamasi yozilsin.
4. $M_1(0;1;3)$ va $M_2(2;4;5)$ nuqtalardan o'tuvchi va Ox o'qqa parallel tekislik tenglamasi yozilsin.
5. Ox o'qdan va $M(0;-2;3)$ nuqtadan o'tuvchi tekislik tenglamasi yozilsin.
6. Oz o'qdan va $M(2;-4;3)$ nuqtadan o'tuvchi tekislik tenglamasi yozilsin.
7. Oy o'qqa parallel, Ox va Oz o'qlardan a va c kesmalar ajratuvchi tekislik tenglamasi yozilsin.
8. $M(2;-1;3)$ nuqtadan o'tuvchi va koordinata o'qlaridan teng kesmalar ajratuvchi tekislik tenglamasi yozilsin.
9. $M(-4;0;4)$ nuqtadan o'tuvchi hamda Ox va Oy o'qlaridan $a=4$ va $b=3$ kesmalar ajratuvchi tekislikning tenglamasi yozilsin.
10. 1) $x-2y+2z-8=0$ va $x+z-6=0$
    2) $x+2z-6=0$ va $x+2y-4=0$
    tekisliklar orasidagi burchak topilsin.
11. $(2;\ 2;\ -2)$ nuqtadan o'tuvchi va $x-2y-3z=0$ tekislikka parallel tekislik topilsin.

12. $(-1; -1; 2)$ nuqtadan o'tuvchi va $x-2y+z-4=0$ hamda $x+2y-2z+4=0$ tekisliklarga perpendikulyar tekislikning tenglamasi yozilsin.
13. $M(-1;2;3)$ nuqtadan OM ga perpendikulyar tekislik tenglamasi yozilsin.
14. Oy o'qdan va $(4; 0; 3)$ nuqtadan o'tuvchi tekislikning tenglamasi yozilsin.
15. Oz o'qqa parallel hamda $M_1(2;2;0)$ va $M_2(4;0;0)$ nuqtalardan o'tuvchi tekislikning tenglamasi yozilsin.
16. $M(1;-3;5)$ nuqtadan o'tuvchi hamda Oy va Oz o'qlardan Ox o'qdagidan ko'ra ikki marta katta kesma ajratuvchi tekislik tenglamasi yozilsin.
17. $(0;0;a)$ nuqtadan o'tuvchi va $x-y-z=0$ hamda $2y=x$ tekisliklarga perpendikulyar tekislikning tenglamasi yozilsin.
18. $M_1(-1;-2;0)$ va $M_2(1;1;2)$ nuqtalardan o'tuvchi hamda $x+2y+2z-4=0$ tekislikka perpendikulyar tekislikning tenglamasi yozilsin.
19. $M_1(1;-1;2)$, $M_2(2;1;2)$ va $M_3(1;1;4)$ nuqtalardan o'tuvchi tekislikning tenglamasi yozilsin.
20. Oz o'qdan $2x+y-\sqrt{5}z=0$ tekislik bilan 60^0 burchak tashkil etuvchi tekislik tenglamasi tuzilsin.
21. $(5;1;-1)$ nuqtadan $x-2y-2z+4=0$ tekislikkacha bo`lgan masofa topilsin.
22. $(4;3;0)$ nuqtadan $M_1(1;3;0)$, $M_2(4;-1;2)$ va $M_3(3;0;1)$ nuqtalardan o'tuvchi tekislikkacha bo'lgan masofa topilsin.
23. $4x+3y-5z-8=0$ va $4x+3y-5z+12=0$ parallel tekisliklar orasidagi masofa topilsin. *Ko'rsatma. Birinchi tekislikda ihtiyoriy, masalan $(2;0;0)$ nuqta olib, undan ikkinchi tekislikkacha bo'lgan masofa topilsin.*
24. $2x-y+3z-9=0$; $x+2y+2z-3=0$; $3x+y-4z+6=0$ tekisliklarning kesishgan nuqtasi topilsin.
25. $(2;-1;1)$ nuqtadan o'tuvchi hamda $3x+2y-z+4=0$ va $x+y+z-3=0$ tekisliklarga perpendikulyar tekislikning tenglamasi yozilsin.

Javoblar:

1. $x+4y-2z=2$
2. $x+y=2a$
3. $x-y+z=a$
4. $2y-3z+7=0$
5. $3y+3z=0$
6. $2x+y=0$
7. $\frac{x}{a}+\frac{z}{c}=1$
8. $x+y+z=4$
9. $\frac{x}{4}+\frac{y}{3}+\frac{z}{2}=1$
10. 1) 45^0; 2)78^030\`
11. $x-2y-3z=4$
12. $2x+3y+4z=3$
13. $x-2y-3z+14=0$
14. $3x-4z=0$
15. $x+y=4$
16. $\frac{x}{2}+\frac{y}{4}+\frac{z}{4}=1$
17. $2x+y+z=a$
18. $2x-2y+z=2$
19. $2x-y+z=5$
20. $3x-y=0$ va $x+3y=0$
21. 3
22. $\sqrt{6}$
23. $2\sqrt{2}$
24. (1;-1;2)
25. $3x-4y+z=11$

15. FAZODA TO'G'RI CHIZIQ TENGLAMASI

1^0. *A(a;b;c)* nuqtadan o'tuvchi va ***P****(m;n;p)* vektorga parallel bo'lgan to'g'ri chiziq tenglamalari. *N(x;y;z)*-to'g'ri chiziqning ixtiyoriy nuqtasi bo'lsin

$$\frac{x-a}{m}=\frac{y-b}{n}=\frac{z-c}{p} \qquad (1)$$

Bu tenglamalar to'g'ri chiziqning *kanonik* tenglamalari deyiladi. ***P****(m;n;p)* vektor to'g'ri chiziqning yo'naltiruvchi vektori deyiladi.

2^0. (1) tenglamadagi har bir nisbatni t parametrga tenglab, to'g'ri chiziqning

$$\begin{cases} x = mt + a \\ y = nt + b \\ z = pt + c \end{cases} \qquad (2)$$

ko'rinishdagi *parametrik tenglamalariga* ega bo'lamiz.

3^0. Ikki nuqta $(x_1;y_1;z_1)$ va $(x_2;y_2;z_2)$ dan o'tuvchi to'g'ri chiziq tenglamalari:

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1} \qquad (3)$$

4^0. To'g'ri chiziqning *umumiy tenglamalari*:

$$\begin{cases} Ax + By + Cz + D = 0 \\ A_1x + B_1y + C_1z + D_1 = 0 \end{cases} \qquad (4)$$

5^0. *Ikki to'g'ri chiziq orasidagi burchak*

$$Cos\varphi = \frac{m\bullet m_1 + n\bullet n_1 + p\bullet p_1}{\sqrt{m^2+n^2+p^2}\bullet\sqrt{m_1^2+n_1^2+p_1^2}} \qquad (5)$$

6^0. $\frac{x-a}{m}=\frac{y-b}{n}=\frac{z-c}{p}$ *to'g'ri chiziq bilan Ax+By+Cz+D=0 tekislik orasidagi burchak*

$$Sin\varphi = \frac{Am+Bn+Cp}{\sqrt{A^2+B^2+C^2}\bullet\sqrt{m^2+n^2+p^2}} \qquad (6)$$

Parallellik sharti: *Am+Bn+Cp=0* (7)

Perpendikulyarlik sharti: $\frac{A}{m}=\frac{B}{n}=\frac{C}{p}$ (8)

7^0. *Tekislik bilan to'g'ri chiziqning kesishgan nuqtasi* (2) ko'rinishidagi to'g'ri chiziqning parametrik tenglamalari tekislikning $Ax+By+Cz+D=0$ tenglamasidagi x, y, z larning t ga nisbatan yozilgan qiymatlarini qo'yamiz. Hosil bo'lgan tenglamadan t_0 ni, so'ngra kesishgan nuqta koordinatalari x_0, y_0, z_0 ni topamiz.

8^0. *Ikki to'g'ri chiziqning bir tekislikda yotish sharti:*

$$\begin{vmatrix} a-a_1 & b-b_1 & c-c_1 \\ m & n & p \\ m_1 & n_1 & p_1 \end{vmatrix} = 0 \qquad (9)$$

Mustaqil yechish uchun masalalar:

1. $x=4, \ y=3$ to'g'ri chiziq yasalsin va uning yo'naltiruvchi vektori topilsin.
2. 1) $\begin{cases} y=3 \\ z=2 \end{cases}$ 2) $\begin{cases} y=2 \\ z=x+1 \end{cases}$ 3) $\begin{cases} x=4 \\ z=y \end{cases}$

   To'g'ri chiziqlar yasalsin va ularning yo'naltiruvchi vektorlari aniqlansin.
3. $A(-1; 2; 3)$ va $B(2; 6; -2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamalari yozilsin va uning yo'naltiruvchi kosinuslari topilsin.
4. $A(2; -1; 3)$ va $B(2; 3; 3)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamalari yozilsin.
5. 1) $(-2; 1; -1)$ nuqtadan o'tuvchi va $P(1; -2; 3)$ vektorga parallel bo'lgan;

   2) $A(3; -1; 4)$ va $B(1; 1; 2)$ nuqtalardan o'tuvchi to'g'ri chiziqning tenglamalari yozilsin.
6. $y=3x-1, \ 2z=-3x+2$ to'g'ri chiziq bilan $2x+y+z-4=0$ tekislik orasidagi burchak topilsin.
7. $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z-1}{3}$ to'g'ri chiziq $2x+y-z=0$ tekislikka parallel ekanligi, $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z+3}{3}$ to'g'ri chiziq esa shu tekislik ustida yotishi ko'rsatilsin.
8. $(-1; 2; -3)$ nuqtadan o'tuvchi va $x=2, \ y-z=1$ to'g'ri chiziqqa perpendikulyar tekislikning tenglamasi yozilsin.

9. $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z+1}{3}$ to’g’ri chiziqdan va *(3; 4; 0)* nuqtadan o’tuvchi tekislikning tenglamasi yozilsin.

10. $\frac{x-1}{1}=\frac{y+1}{2}=\frac{z+2}{2}$ to’g’ri chiziqdan o’tuvchi va *2x+3y-z=4* tekislikka perpendikulyar tekislikning tenglamasi yozilsin.

11. (*a; b; c*) nuqtadan o’tuvchi va: 1) *Oz* o’qqa parallel; 2) *Oz* o‘qqa perpendikulyar bo’lgan to’g’ri chiziq tenglamalari yozilsin.

12. $\begin{cases} x=2z-1 \\ y=-2z+1 \end{cases}$ to’g’ri chiziq bilan *(1; -1; -1)* nuqta va koordinatalar boshidan o’tuvchi to’g’ri chiziq orasidagi burchak topilsin.

13. *(2; -3; 4)* nuqtadan *Oz* o’qqa tushirilgan perpendikulyarning tenglamalari yozilsin.

Ko’rsatma. Izlangan to’g’ri chiziq (0; 0; 4) nuqtadan ham o’tadi.

14. *N(2; -3; 4)* nuqtadan $\frac{x+1}{3}=\frac{y+2}{4}=\frac{z-1}{5}$ to’g’ri chiziqqacha bo’lgan masofa topilsin.

Ko’rsatma. A(-1; -2; 1) - to’g’ri chiziqdagi nuqta; $P(3;4;5)$*- to’g’ri chiziqning yo’naltiruvchi vektori. U vaqtda*

$$d=AN\sin\alpha=\frac{AN\left|P\bullet\overline{AN}\right|}{P\bullet AN}=\frac{\left|P\bullet\overline{AN}\right|}{P}$$

15. $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z+3}{2}$ va $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z+1}{2}$ parallel to’g’ri chiziqlar orasidagi masofa topilsin.

16. $\begin{cases} 2x+y+8z-16=0 \\ x-2y-z+2=0 \end{cases}$ to’g’ri chiziq tenglamalari:

1) proyektsiyalari bo’yicha; 2)kanonik ko’rinishda yozilsin. To’g’ri chiziqning koordinatalar tekisliklaridagi izlari topilsin, to’g’ri chiziq va uning proyektsiyalari yasalsin.

17. *A(0; -4; 0)* nuqtadan o'tuvchi va $P(1;\,2;\,3)$ vektorga parallel to'g'ri chiziq tenglamalari yozilsin; to'g'ri chiziqning *xOz* tekisligidagi izi topilsin.

18. x=3, z=5 to'g'ri chiziqning yo'naltiruvchi vektori topilsin.

19. *(2; -3; 4)* nuqtadan *Oy* o'qqa tushirilgan perpendikulyarning tenglamalari yozilsin.

20. $\begin{cases} 2x - y - 7 = 0 \\ 2x - z + 5 = 0 \end{cases}$ *va* $\begin{cases} 3x - 2y + 8 = 0 \\ z = 3x \end{cases}$

to'g'ri chiziqlar orasidagi burchak topilsin.

21. $\frac{x-3}{2} = \frac{y}{1} = \frac{z-1}{2}$ va $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z}{2}$ parallel to'g'ri chiziqlardan o'tuvchi tekislikning tenglamasi yozilsin.

22. *x=2t-1, y=t+2, z=1-t* to'g'ri chiziqning *3x-2y+z=3* tekislik bilan kesishgan nuqtasi topilsin.

23. $\frac{x}{2} = \frac{y-1}{1} = \frac{z+1}{2}$ to'g'ri chiziqning *x+2y+3z-29=0* tekislik bilan kesishgan nuqtasi topilsin.

24. $\left.\begin{matrix} x = z - 2 \\ y = 2z + 1 \end{matrix}\right\}$ va $\frac{x-2}{3} = \frac{y-4}{1} = \frac{z-2}{1}$

to'g'ri chiziqlarning kesishuvchi ekanligi ko'rsatilsin va ular yotgan tekislikning tenglamasi yozilsin.

25. *(2; 1; 0)* nuqtadan *x=3z-1; y=2z* to'g'ri chiziqqa tushirilgan perpendikulyarning tenglamalari yozilsin.

Javoblar:

1. $P(0;0;1)$
2. 1) $\overline{P} = \overline{i}$ 2) $\overline{P} = \overline{i} + \overline{k}$ 3) $\overline{P} = \overline{j} + \overline{k}$
3. $\frac{x+1}{2} = \frac{y-2}{4} = \frac{z-3}{-5}$: $Cos\alpha = 0{,}3\sqrt{2}$; $Cos\beta = 0{,}4\sqrt{2}$; $Cos\gamma = -0{,}5\sqrt{2}$
4. x=2; z=3

5. 1) $\begin{cases} x = -2+t \\ y = 1-2t \\ z = -1+3t \end{cases}$ 2) $\begin{cases} x = 1+t \\ y = 1-t \\ z = 2+t \end{cases}$ 6. $Sin\varphi = \frac{1}{\sqrt{6}}$

7. Ikkala to'g'ri chiziq uchun ham *Am+Bn+Cp*=2•2+1•(-1)+(-1)•3=0, lekin birinchisining (-1;-1;1) nuqtasi tekislikda yotmaydi, ikkinchisining (-1;-1;-3) nuqtasi esa tekislikda yotadi.

8. *y+z+1=0* *(to'g'ri chiziqning tenglamalarini* $\frac{x-2}{0} = \frac{y-1}{1} = \frac{z}{1}$ *ko'rinishda yozish mumkin).*

9. *x-2y+z+5=0*

10. *8x-5y+z-11=0*

11. 1) $\frac{x-a}{0} = \frac{y-b}{0} = \frac{z-c}{1}$, demak, $\begin{cases} x = a \\ y = b \end{cases}$

2) *z=c* va $\frac{x-a}{m} = \frac{y-b}{n}$

12. $Cos\varphi = \frac{1}{\sqrt{3}}$

13. *3x+2y=0 z=4*

14. $0{,}3\sqrt{38}$

15. $\frac{4\sqrt{2}}{3}$

16. $\begin{cases} x = 6 - 3z \\ y = -2z+4 \end{cases}$ $\frac{x-6}{-3} = \frac{y-4}{-2} = \frac{z}{1}$ izlari: (6;4;0), (0;0;2)

17. $\frac{x}{1} = \frac{y+4}{2} = \frac{z}{3}$

18. *P(0;1;0)*

19. *y=-3; 2x-z=0*

20. Tenglamalarni kanonik formaga keltiramiz;

$\frac{x}{1} = \frac{y+7}{2} = \frac{z-5}{2}$ va $\frac{x}{2} = \frac{y-4}{3} = \frac{z}{6}$; $Cos\varphi = \frac{20}{21} \approx 0{,}952$; $\varphi = 17^0 48`$

21. *x+2y-2z=1*

22. (5; 5; -2)

23. (6; 4; 5)

24. *x+2y-5z=0*

25. $\frac{x-2}{-9} = \frac{y-1}{8} = \frac{z}{11}$

16. CHIZIQLI FAZO. EVKLID FAZO. ORTOGONAL MATRITSA

1. $\boldsymbol{a}_1(0;\ 1;\ -3)$, $\boldsymbol{a}_2(3;\ 5;\ 0)$, $\boldsymbol{a}_3(1;\ 2;\ -1)$ vektorlar sistemalariga tortilgan chiziqli qism osti fazosining bazislaridan birini, o`lchamini hamda ortonormallangan bazisini topamiz:

Buning uchun $\boldsymbol{a}_1x_1+\boldsymbol{a}_2x_2+\boldsymbol{a}_3x_3=\theta$ vektor tenglama umumiy yechimini Gauss-Jordan usulida quramiz:

$$\begin{pmatrix} 0 & 3 & 1 & | & 0 \\ 1 & 5 & 2 & | & 0 \\ -3 & 0 & -1 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 3 & 1 & | & 0 \\ 1 & 5 & 2 & | & 0 \\ 0 & 15 & 5 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 3 & 1 & | & 0 \\ 1 & -1 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

x_1, x_3 noma'lumlar umumiy yechimning bazis noma'lumlari. Demak, mos ravishda, $\boldsymbol{a}_1$, $\boldsymbol{a}_3$ vektorlar tizimi berilgan sistemaning bazislaridan birini tashkil etadi. Tizim 2 ta vektordan tarkib topgani uchun, berilgan vektorlar sistemasining o`lchami 2 ga teng.

Bazisni tashkil qiluvchi $\boldsymbol{a}_1(0;\ 1;\ -3)$ va $\boldsymbol{a}_3(1;\ 2;\ -1)$ vektorlarni ortogonallaymiz:

$$b_1 = a_1(0;1;-3)$$

$$b_2 = a_3 - \frac{(b_1 \cdot a_3)}{(b_1 \cdot b_1)} b_1 = (1;2;-1) - \frac{0\cdot 1 + 1\cdot 2 + (-3)\cdot(-1)}{0\cdot 0 + 1\cdot 1 - 3\cdot(-3)}(0;1;-3) = (1;2;-1) - \frac{5}{10}(0;1;-3) = (1;\frac{3}{3};\frac{1}{2})$$

hosil bo`lgan ortogonal sistema vektorlarini butun koordinatali vektorlarga aylantirib, $\boldsymbol{b}_1(0;\ 1;\ -3)$ va $\boldsymbol{b}_2(2;\ 3;\ 1)$ ni olamiz. Bu ortogonal sistemaning har bir vektorini birlik ko`rinishga keltiramiz, ya'ni ortonormallashtiramiz:

$$\frac{b_1}{|b_1|} = \frac{(0;1;-3)}{\sqrt{0^2+1^2+(-3)^2}} = (0;\frac{1}{\sqrt{10}};-\frac{3}{\sqrt{10}})$$

$$\frac{b_2}{|b_2|} = \frac{(2;3;1)}{\sqrt{2^2+3^2+1^2}} = \left(\frac{2}{\sqrt{14}};\frac{3}{\sqrt{14}};\frac{1}{\sqrt{14}}\right)$$

2. $\boldsymbol{x}(3;\ -2;\ 4)$ vektor $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$ bazisda berilgan. Vektorning

$$\begin{cases} e_1' = e_1 + 2e_2 - 3e_3 \\ e_2' = e_1 + e_2 + e_3 \\ e_3' = 2e_1 - e_2 + 2e_3 \end{cases}$$

bazisdagi koordinatalarini topamiz:

Koeffitsientlar matritsasi *P* ning transponirlangan matritsasi P^T ni hosil qilamiz:

$$P = \begin{pmatrix} 1 & 2 & -3 \\ 1 & 1 & 1 \\ 2 & -1 & 2 \end{pmatrix} \qquad P^T = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & -1 \\ -3 & 1 & 2 \end{pmatrix}$$

U holda ***x*** vektorning dastlabki bazisdagi koordinatalari uning yangi bazisdagi koordinatalari orqali (matritsa shaklida $x=P^T x`$) quyidagicha ifodalanadi:

$$\begin{cases} x_1 = x_1^` + x_2^` + 2x_3^` \\ x_2 = 2x_1^` + x_2^` - x_3^` \\ x_3 = -3x_1^` + x_2^` + 2x_3^` \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 2 & 1 & -1 & -2 \\ -3 & 1 & 2 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & -1 & -5 & 8 \\ 0 & 4 & 8 & 13 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -3 & -5 \\ 0 & 1 & 5 & 8 \\ 0 & 0 & -12 & -19 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1/4 \\ 0 & 1 & 0 & 1/12 \\ 0 & 0 & 1 & 19/12 \end{array}\right)$$

Demak, dastlab berilgan ***x****(3; -2; 4)* vektorning yangi bazisdagi koordinatalari: $\boldsymbol{x}\left(-\frac{1}{4}; \frac{1}{12}; \frac{19}{12}\right)$

Ta'rif: $P \cdot P^T = P \cdot P^{-1} = E$ shartni bajaruvchi *P* matritsaga ortogonal matritsa deyiladi.

3. Quyidagi matritsa ortogonal matritsa bo'lishini tekshiramiz :

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & Cos\alpha & -Sin\alpha \\ 0 & Sin\alpha & Cos\alpha \end{pmatrix} \qquad P^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & Cos\alpha & Sin\alpha \\ 0 & -Sin\alpha & Cos\alpha \end{pmatrix}$$

$$P \cdot P^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E$$

Demak, berilgan P matritsa ortogonal matritsa bo'ladi.

Mustaqil yechish uchun masalalar:

Quyidagi vektorlar sistemalariga tortilgan chiziqli qism osti fazosining bazislaridan birini , o'lchamini va ortonormallangan bazisini toping:

4. $\boldsymbol{a}_1(3; -1; 2)$, $\boldsymbol{a}_2(1;4;-1)$, $\boldsymbol{a}_3(7;2;3)$

5. $\boldsymbol{x}(2;-1)$ vektor $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, bazisda berilgan . Vektorning $\boldsymbol{e}_1`=\boldsymbol{e}_1-3\boldsymbol{e}_2$; $\boldsymbol{e}_2`=2\boldsymbol{e}_1+\boldsymbol{e}_2$ bazisdagi koordinatalarini toping .

Quyidagi matritsalardan ortogonallarini ajrating:

6. $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 0.5 \\ 4 & -1 & 3 \end{pmatrix}$ 7. $\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$ 8. $\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$

Quyida berilgan ikki vektorlar sistemalaridan har biri bazis bo'la olishini isbotlang. Ushbu bazislarda berilgan aynan bir vektorning koordinatalari orasida munosabatlarni o'rnating:

9. $\boldsymbol{e}_1(1;2)$, $\boldsymbol{e}_2(1;1)$;
$\boldsymbol{e}_1`(1;1)$, $\boldsymbol{e}_2`(3;4)$

10. R_3 da $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ bazisdan fazoni Oy ordinata o'qi atrofida α burchakka burgandagi bazisga o'tish matritsasini quring.

Quyidagi vektorlar sistemalariga tortilgan chiziqli qism osti fazosining bazislaridan birini , o'lchamini va ortonormallangan bazisini toping:

11. $\boldsymbol{a}_1(1;2;-1;3)$, $\boldsymbol{a}_2(0;3;4;1)$, $\boldsymbol{a}_3(-2;-1;6;-5)$, $\boldsymbol{a}_4(5;4;2;-4)$

12. $\boldsymbol{x}(3;-2)$ vektor $\boldsymbol{e}_1$, $\boldsymbol{e}_2$ bazisda berilgan vektorning $\boldsymbol{e}_1`=2\boldsymbol{e}_1-\boldsymbol{e}_2$; $\boldsymbol{e}_2`=\boldsymbol{e}_1+\boldsymbol{e}_2$ bazisdagi koordinatalarini toping .

13. $\boldsymbol{x}(1;2;-2)$ vektor $\boldsymbol{e}_1$, $\boldsymbol{e}_2$ bazisda berilgan vektorning $\boldsymbol{e}_1`=\boldsymbol{e}_1+\boldsymbol{e}_2-\boldsymbol{e}_3$; $\boldsymbol{e}_2`=2\boldsymbol{e}_1-\boldsymbol{e}_2+\boldsymbol{e}_3$ bazisdagi koordinatalarini toping .

Quyidagi matritsalardan ortogonallarini ajrating:

14. $\begin{pmatrix} Sin\alpha & 0 & Cos\alpha \\ 0 & 1 & 1 \\ -Cos\alpha & 0 & Sin\alpha \end{pmatrix}$ 15. $\begin{pmatrix} tg\alpha & tg\alpha \\ -ctg\alpha & ctg\alpha \end{pmatrix}$

Quyida berilgan ikki vektorlar sistemalaridan har biri bazis bo'la olishini isbotlang. Ushbu bazislarda berilgan aynan bir vektorning koordinatalari orasida munosabatlarni o'rnating:

16. ***e**$_1$(2;1;-1),* ***e**$_2$(3;1;2),* ***e**$_3$(1;0;4)*

***e**$_1$`(1;1;-1),* ***e**$_2$`(2;3;-2),* ***e**$_3$`(3;4;-4)*

Javoblar:

4. Bazislaridan biri ***a**$_1$;* ***a**$_2$;* o'lchami 2;

$$\frac{b_1}{|b_1|}=\left(\frac{3}{\sqrt{14}};-\frac{1}{\sqrt{14}};\frac{2}{\sqrt{14}}\right);\quad \frac{b_2}{|b_2|}=\left(\frac{5}{\sqrt{254}};\frac{15}{\sqrt{254}};-\frac{2}{\sqrt{254}}\right)$$

5. $x\left(-\frac{4}{5};\frac{7}{5}\right)$

6. Ortogonal emas.

7. Ortogonal emas.

8. Ortogonal emas.

9. ***e**$_1$`=**e**$_2$;* ***e**$_2$`=**e**$_1$+2**e**$_2$*

10. $P=\begin{pmatrix} Cos\alpha & 0 & Sin\alpha \\ 0 & 1 & 0 \\ Sin\alpha & 0 & -Cos\alpha \end{pmatrix}$

11. Bazislaridan biri ***a**$_1$, **a**$_2$, **a**$_4$*; o'lchami 3;

$$\frac{b_1}{|b_1|}=\left(\frac{1}{\sqrt{15}};\frac{2}{\sqrt{15}};-\frac{1}{\sqrt{15}};\frac{3}{\sqrt{15}}\right)$$

$$\frac{b_2}{|b_2|}=\left(-\frac{1}{\sqrt{219}};\frac{7}{\sqrt{219}};\frac{13}{\sqrt{219}};0\right)$$

$$\frac{b_3}{|b_3|}=\left(\frac{15909}{|b_3|};\frac{18723}{|b_3|};\frac{15906}{|b_3|};-\frac{12483}{|b_3|}\right)\qquad |b_3|=\sqrt{1022473135}$$

12. $x\left(\frac{5}{3};-\frac{1}{3}\right)$

13. $x\left(\frac{5}{3};-\frac{1}{3};0\right)$

14. Ortogonal.

15. Ortogonal emas.

16. $$\begin{cases} e_1^{`}=-3e_1+4e_2-3e_3 \\ e_2^{`}=20e_1-17e_2+13e_3 \\ e_3^{`}=24e_1-20e_2+15e_3 \end{cases}$$

17. CHIZIQLI OPERATOR

1. Agar R^3da chiziqli $\widetilde{A}$ operator $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda o'zining $A = \begin{bmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{bmatrix}$ matritsasi bilan berilgan bo'lsa, $x = 4\vec{e}_1 - 3\vec{e}_2 + \vec{e}_3$ vektorning *y=A(x)* aksini toping.

Y=AX formulaga binoan, $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{bmatrix} * \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ -13 \\ -18 \end{bmatrix}$

Demak, $y = 10e_1 - 13e_2 - 18e_3$

2. $\vec{e}_1, \vec{e}_2$ bazisda $\widetilde{A}$ operator $A = \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}$ matritsaga ega.

$e_1 = e_1 - 2e_2, \quad e_2 = 2e_2 + e_2$ bazisida $\widetilde{A}$ operatorining matritsasini toping.

O'tish matritsasi $C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$ ning teskari matrisasi $C^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

Demak, $B = C^{-1}AC = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}$

3. Chizqli $\widetilde{A}$ operator $A = \begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

Xarakteristik tenglama tuzamiz:

$$|A - \lambda E| = \begin{vmatrix} 1-\lambda & 4 \\ 9 & 1-\lambda \end{vmatrix} = 0, \quad \lambda^2 - 2\lambda - 35 = 0; \quad \lambda_1 = -5, \quad \lambda_2 = 7$$

λ=-5 ga tegishli $X^{(1)}=(X_1,X_2)$ xos vektorni topamiz. Buning uchun quyidagi tenglamani echamiz:

$$\lambda = -5 \quad (A - \lambda E) \cdot x = 0 \qquad \begin{pmatrix} 6 & 4 \\ 9 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow X_2 = -1{,}5X_1$$

agar $X_1=C$ deb olsak $X_2=-1.5C$, $X^{(1)}=(C;-1.5C)$ vektorlar har qanday $C\neq0$ uchun A operatorini xos qiymati $\lambda=5$ ga tegishli xos vektor bo'ladi. Huddi shunday $\lambda_2=7$ xos qiymati uchun A operatorni xos vektorlarni $X^{(2)}=\left(\frac{2}{3}C_1, C_1\right)$, $C_1\neq 0$ vektorlar tashkil etadi.

4. Chiziqli operatorning $A=\begin{pmatrix}1 & 4\\ 9 & 1\end{pmatrix}$ matritsasini diogonal ko'rinishiga keltiring.

$A=\begin{pmatrix}1 & 4\\ 9 & 1\end{pmatrix}$ matrisa bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlari 3-misolda topilgan: $\lambda=-5$ $\lambda_2=7$

$X^{(1)}=(C;1{,}5C)$; $X^{(2)}=\left(\frac{2}{3}C_1, C_1\right)$; $X^{(1)}$ *va* $X^{(2)}$ vektorning koordinatalari proportsional emas, shuning uchun $X^{(1)}$ va $X^{(2)}$ vektorlar chiziqli erkli. Demak, $X^{(1)}$ va $X^{(2)}$ bazisda A-matritsaning diagonal ko'rinishi:

$A^*=\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}$ *ёки* $A^*=\begin{pmatrix}5 & 0\\ 0 & 7\end{pmatrix}$. Buni tekshirish uchun bazis vektorlar sifatida $X^{(1)}=(2;\ -3)$, $X^{(2)}=(4;\ 6)$ vektorlarni olsak, yangi bazisga o'tkazuvchi o'tish matritsa C ning ko'rinish: $\begin{pmatrix}2 & 4\\ -3 & 6\end{pmatrix}$ bo'ladi. Diogonal matrtisa:

$$A^*=C^{-1}AC=\begin{pmatrix}2 & 4\\ -3 & 6\end{pmatrix}^{-1}\begin{pmatrix}1 & 4\\ 9 & 1\end{pmatrix}\begin{pmatrix}2 & 4\\ -3 & 6\end{pmatrix}=\frac{1}{24}\begin{pmatrix}6 & -4\\ 3 & 2\end{pmatrix}\begin{pmatrix}1 & 4\\ 9 & 1\end{pmatrix}\begin{pmatrix}2 & 4\\ -3 & 6\end{pmatrix}=\frac{1}{24}\begin{pmatrix}-30 & 20\\ 21 & 14\end{pmatrix}$$

$$\begin{pmatrix}2 & 4\\ -3 & 6\end{pmatrix}=\frac{1}{24}\begin{pmatrix}-100 & 0\\ 0 & 168\end{pmatrix}=\begin{pmatrix}-5 & 0\\ 0 & 7\end{pmatrix}$$

Mustaqil yechish uchun masalalar:

5. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli $\widetilde{A}$ operator $A=\begin{pmatrix}3 & 2\\ -1 & 5\end{pmatrix}$ matritsa bilan berilgan, $x=4e_1-3e_2$ bo'lsa, $y=A(x)$ ni toping.

6. $\vec{e}_1,\vec{e}_2,\vec{e}_3$ bazisda chiziqli $\widetilde{A}$ operator $\begin{pmatrix} -1 & 0 & 2 \\ 2 & 1 & 3 \\ 3 & 0 & 1 \end{pmatrix}$ matritsa bilan berilgan $x=2e_1-4e_2-e_3$ bo'lsa, $y=A(x)$ ni toping.

7. $\vec{e}_1,\vec{e}_2$ bazisda $\widetilde{A}$ operatorlar $A=\begin{pmatrix} 2 & 4 \\ -3 & 3 \end{pmatrix}$ matritsaga ega. $e_1^* = e_2 - 2e_1,\quad e_2^* = 2e_1 - 4e_2$ bazisda $\widetilde{A}$ operatorning matritsasini toping.

Berilgan matritsalarning xos qiymatlari va xos vektorlarini toping:

8. $A=\begin{pmatrix} 2 & 4 \\ -1 & -3 \end{pmatrix}$

9. $A=\begin{pmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ 1 & 3 & 0 \end{pmatrix}$

10. $A=\begin{pmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{pmatrix}$

11. $\vec{e}_1,\vec{e}_2,\vec{e}_3$, bazisdan $\vec{e}_2,\vec{e}_3,\vec{e}_1$ bazisga o'tish matrisasini toping.

12. $\vec{e}_1,\vec{e}_2,\vec{e}_3,e_4$ bazisda $\widetilde{A}$ operatorining matritsasi

$A=\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 3 & -1 & 2 \\ 2 & 5 & 0 & 1 \\ 1 & 2 & 1 & 3 \end{pmatrix}$ berilgan. Ushbu operatorning

1) $\vec{e}_1,\vec{e}_3,\vec{e}_2,e_4$ bazisdagi matritsasini toping;

2) $e_1 \cdot e_1 + e_2 \cdot e_1 + e_2 + e_3 \cdot e_1 + e_2 + e_3 + e_4$ bazisdagi matritsasini toping.

O'zlarining matritsalari bilan berilgan chiziqli operatorlarning xos qiymatlari va xos vektorlarini toping:

13. $A=\begin{pmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{pmatrix}$;

14. $A=\begin{pmatrix} 4 & -5 & 2 \\ 5 & -7 & 3 \\ 6 & -9 & 4 \end{pmatrix}$;

15. $A=\begin{pmatrix}1 & -3 & 3\\ 2 & 6 & 3\\ -1 & -4 & 8\end{pmatrix}$; 16. $A=\begin{pmatrix}7 & -12 & 6\\ 10 & -19 & 10\\ 12 & -24 & 13\end{pmatrix}$;

17. $A=\begin{pmatrix}1 & 0 & 0\\ 1 & 2 & 1\\ -1 & 0 & 1\end{pmatrix}$

Chiziqli operatorning $\widetilde{A}$ matritsasini diogonal ko'rinishiga keltiring:

18. $A=\begin{pmatrix}1 & 2 & 0\\ 0 & 2 & 0\\ -2 & -2 & 1\end{pmatrix}$ 19. $A=\begin{pmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -6 & 1 & 7 & -1\end{pmatrix}$

Javoblar:

5. $6\overline{e}_1-19\overline{e}_2$ 6. $-4\overline{e}_1+7\overline{e}_2+7\overline{e}_3$

7. $\begin{pmatrix}-3 & 14\\ -3 & 8\end{pmatrix}$

8. $(4c_1-c),\quad \lambda=1\qquad (c_1-c_1),\quad \lambda=-2$

9. $(-2c,\,c,\,c),\quad \lambda_1=1,\ (0,\,c_1,\,c_1),\quad \lambda_2=3\quad (6c_2,-7c_2,5c_2),\ \lambda_3=-3$

10. $(c,\,c,\,-c),\quad \lambda=-1$ 11. $\begin{pmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}$;

12. 1) $\begin{pmatrix}1 & 0 & 2 & 1\\ 2 & 0 & 5 & 1\\ 0 & -1 & 3 & 2\\ 1 & 1 & 2 & 3\end{pmatrix}$ 2) $\begin{pmatrix}-2 & 0 & 1 & 0\\ 1 & -4 & -8 & -7\\ 1 & 4 & 6 & 1\\ 1 & 3 & 4 & 7\end{pmatrix}$

13. $\lambda=2,\ x=(c_1,\,2c_1,\,c_2)$;

14. $\lambda_1=1,\quad x=(c,\,c,\,c);\quad \lambda_2=0\quad x=(c,\,2c,\,3c)$

15. $\lambda_1=1,\ x(3c,\,c,\,c)$;

16. $\lambda_1=1,\ x=(2c_1+c_2;\ c_1-c_2);\ \lambda_2=-1,\ x=(3c,\,5c,\,0)$

17. $\lambda_1=2,\ x=(0,\,c_1,\,0)\quad \lambda_2=-1,\ x(0,\,c_2,\,-c_2)$

18. $\begin{pmatrix}2 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{pmatrix}$ 19. $\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 3\end{pmatrix}$

18. KVADRATIK FORMALAR

1. $L(x_1, x_2, x_3) = 4x_1^2 - 12x_1x_2 - 10x_1x_3 + x_2^2 - 3x_3^2$ kvadratik formaning A matritsasini tuzing.

Kvadratik formaning matritsasini topamiz:

$$L = (x_1, x_2, x_3)\begin{pmatrix} 4 & -6 & -5 \\ -6 & 1 & 0 \\ -5 & 0 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad A = \begin{pmatrix} 4 & -6 & -5 \\ -6 & 1 & 0 \\ -5 & 0 & -3 \end{pmatrix}$$

2. $L(x_1, x_2) = 2x_1^2 + 4x_1x_2 - 3x_2^2$ kvadratik forma berilgan.

$x_1 = 2y_1 - 3y_2;\ x_2 = y_1 + y_2;$ chiziqli almashtirish orqali hosil bo'lgan $L(y_1, y_2)$ kvadratik formani toping.

Berilgan kvadratik formaning matritsasi $A = \begin{pmatrix} 2 & 2 \\ 2 & -3 \end{pmatrix}$ chiziqli almashtirish matritsasi $C = \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}$ bo'ladi.

Qidirilayotgan kvadratik formaning matritsasi quyidagicha:

$$A' = \begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 2 & 2 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 13 & -17 \\ -17 & 3 \end{pmatrix}$$

kvadratik formaning ko'rinishi:

$L(y_1, y_2) = 13y_1^2 - 34y_1y_2 + 3y_2^2$

3. Kvadratik formani – kanonik ko'rinishga keltiring:

$L(x_1, x_2, x_3) = x_1^2 - 3x_1x_2 + 4x_1x_3 + 2x_2x_3 + x_3^2 = x_1^2 - x_1(3x_2 - 4x_3) + 2x_2x_3 + x_3^2$

x_1 o'zgaruvchining kvadrati o'rnida turgan koeffitsiyenti no'ldan farqli bo'lgani uchun, x_1 o'zgaruvchining to'liq kvadratini topamiz:

$$L = \left[x_1 - 2x_1\left(\frac{1}{2}(3x_2 - 4x_3)\right) + \left(\frac{1}{2}(3x_2 - 4x_3)\right)^2\right] - \left(\frac{1}{2}(3x_2 - 4x_3)\right)^2 + 2x_2x_3 + x_3^2 =$$

$$\left(x_1 - \frac{3}{2}x_2 + 2x_3\right)^2 - \frac{9}{4}x_2^2 + 8x_2x_3 - 3x_3^2$$

endi o'zgaruvchi x_2 uchun kvadratini topamiz:

$$L = \left(x_1 - \frac{3}{2}x_2 + 2x_3\right)^2 - \frac{9}{4}\left(x_2 - \frac{16}{9}x_3\right)^2 + \frac{37}{9}x_3^2,$$

Demak, no'ldan farqli chiziqli almashtirish

$$y_1 = x_1 - \frac{3}{2}x_2 - 2x_3$$

$$y_2 = x_2 - \frac{16}{9}x_3$$

$y_3=y_3$ berilgan kvadratik formani kanonik ko'rinishga keltiradi:

$$L(y_1,\ y_2,\ y_3) = y_1^2 - \frac{9}{4}y_2^2 + \frac{37}{9}y_3^2$$

4. Kvadratik forma $L=13x_1^2-6x_1x_2+5x_2^2$ musbat aniqlangan kvadratik forma ekanligini isbotlang:

Kvadratik formaning matritsasi $A = \begin{pmatrix} 13 & -3 \\ -3 & 5 \end{pmatrix}$ bo'ladi.

Xarakteristik tenglama tuzamiz:

$$|A - \lambda E| = \begin{vmatrix} 13-\lambda & -3 \\ -3 & 5-\lambda \end{vmatrix} \quad \text{yoki} \quad \lambda^2 - 18\lambda + 56 = 0$$

ya'ni $\lambda_1 = 14,\ \ \lambda_2 = 4$ xarakteristik tenglamaning yechimlari musbat bo'lgani uchun, L-musbat aniqlangan kvadratik forma bo'ladi.

Mustaqil yechish uchun masalalar:

5. Kvadratik formani matritsa ko'rinishida yozing:

$$L = 2x_1^2 + 3x_2^2 - x_3^2 + 4x_1x_2 - 6x_1x_3 + 10x_2x_3$$

6. Kvadratik formaning matritsasini toping:

$$L(x_1,\ x_2,\ x_3) = (x_1,\ x_2,\ x_3)\begin{pmatrix} -1 & 0 & 2 \\ 2 & 4 & 1 \\ 3 & 0 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

7. Kvadratik forma $L(x_1,\ x_2) = 3x_1^2 - x_2^2 + 4x_1x_2$, berilgan.

$x_1=2y_1-y_2,\ \ x_2=y_1-y_2,$ chiziqli almashtirish orqali hosil bo'lgan kvadratik formani toping.

Kvadratik formani qanday aniqlanganligini toping:

8. $x_1^2 + 4x_2^2 + 3x_3^2 + 2x_1x_2$

9. $-2x_2^2 - x_1^2 - x_1x_3 + 2x_2x_3 - 2x_3^2$

10. $x_1^2 + 26x_2^2 + 10x_1x_2$

11. $-x_1^2+2x_1x_2-4x_2^2$

12. $x_1^2+15x_2^2+4x_1x_2-2x_1x_3+6x_2x_3$

13. $12x_1x_2-12x_1x_3+6x_2x_3-11x_1^2-6x_2^2-6x_3^2$

14. $x_1^2+4x_2^2+4x_3^2+8x_4^2+8x_2x_4$

Kvadratik formani kanonik ko'rinishga keltiring:

15. $3x_2^2+3x_3^2+4x_1x_2+4x_1x_3-2x_2x_3$

16. $7x_1^2+7x_2^2+7x_3^2+2x_1x_2+2x_1x_3-2x_2x_3$

17. $x_1x_2+x_1x_3+x_2x_3$

18. $17x_1^2+14x_2^2+14x_3^2-4x_1x_2-4x_1x_3-8x_2x_3$

Javoblar:

5. $L=(x_1, x_2, x_3) \begin{pmatrix} 2 & 2 & -3 \\ 2 & 3 & 5 \\ -3 & 5 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

6. $\begin{pmatrix} -1 & 1 & 2.5 \\ 1 & 4 & 0.5 \\ 2.5 & 0.5 & -1 \end{pmatrix}$

7. $L = (y_1, y_2) = 19y_1^2 - 22y_1y_2 + 6y_2^2$

8. Musbat aniqlangan.

9. Manfiy aniqlangan.

10. Musbat aniqlangan.

11. Manfiy aniqlangan.

12. Umumiy ko'rinishda.

13. Manfiy aniqlangan.

14. Musbat aniqlangan.

15. $4y_1^2 + 4y_2^2 - 2y_3^2$

16. $8y_1^2 + 8y_2^2 + 5y_3^2$

17. $y_1^2 + y_2^2 - y_3^2$

18. $9y_1^2 + 18y_2^2 + 18y_3^2$

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