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«МАТЕМАТИКА» КАФЕДРАСИ

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4. MATRITSA RANGINI HISOBLASH.

TESKARI MATRITSANI TOPISH

1. $A=\begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{pmatrix}$ (1)

A matritsaning rangi deb noldan farqli minorlarning eng yuqori tartibiga aytiladi va *rang(A)* kabi ifodalanadi.

Matritsa rangi ikki usulda topiladi:

1. Matritsa rangi ta'rifga asoslangan "minorlar ajratish" usuli.
2. Elementar almashtirishlar bajarib diagonal matritsaga keltirishga asoslangan "Gauss algoritmi".

Misol 1. Matritsa rangini hisoblang:

$A=\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}$ *A* matritsa 3x5 tartibli, demak uning rangi 3 dan yuqori bo'lmaydi. Uchinchi tartibli minorlarni hisoblaymiz:

$M_1=\begin{vmatrix} 2 & -1 & 3 \\ 4 & -2 & 5 \\ 2 & -1 & 1 \end{vmatrix}$=-4-10-12+12+4+10=0 $M_2=\begin{vmatrix} 2 & -1 & -2 \\ 4 & -2 & 1 \\ 2 & -1 & 8 \end{vmatrix}$=-32-2+8-8+32+2=0

$M_3=\begin{vmatrix} 2 & -1 & 4 \\ 4 & -2 & 7 \\ 2 & -1 & 2 \end{vmatrix}$=-8-14-16+16+8-14=0 $M_4=\begin{vmatrix} -1 & 3 & -2 \\ -2 & 5 & 1 \\ -1 & 1 & 8 \end{vmatrix}$=-40-3+4-10+48+1=0

$M_5=\begin{vmatrix} -1 & 3 & 4 \\ -2 & 5 & 7 \\ -1 & 1 & 2 \end{vmatrix} = -10-21-8+20+12+7=0$ $M_6=\begin{vmatrix} 3 & -2 & 4 \\ 5 & 1 & 7 \\ 1 & 8 & 2 \end{vmatrix}$=6-14+160-4+20-168=0

$M_7=\begin{vmatrix} -1 & -2 & 4 \\ -2 & 1 & 7 \\ -1 & 8 & 2 \end{vmatrix} = -2+14-64+4+56-8=0$

$M_8=\begin{vmatrix} 2 & 3 & 4 \\ 4 & 5 & 7 \\ 2 & 1 & 2 \end{vmatrix} = 20+42+16-40-14-24=0$

$$M_9=\begin{vmatrix}2 & 3 & -2\\ 4 & 5 & 1\\ 2 & 1 & 8\end{vmatrix}=80+6-8+20-2-96=0$$

$$M_{10}=\begin{vmatrix}2 & -2 & 4\\ 4 & 1 & 7\\ 2 & 8 & 2\end{vmatrix}=4+128-28-8+16-112=0$$

Barcha uchinchi tartibli minorlar nolga teng. Ikkinchi tartibli minorlarni hisoblaymiz:

$M_1^1=\begin{vmatrix}-1 & 3\\ -2 & 5\end{vmatrix}=-5+6=1$ $\quad M_1^1 \neq 0 \quad r(A)=2$

Bu usulda noldan farqli minor topilgunga qadar hisoblashlar davom etadi. Shuning uchun tartibi kattaroq matritsa rangini hisoblash bir muncha qiyinchiliklarga olib keladi.

Misol 2. Matritsa rangini elementarni almashtirishlar yordamida nollar yig'ib hisoblaymiz:

$$A=\begin{pmatrix}25 & 31 & 17 & 43\\ 75 & 94 & 53 & 132\\ 75 & 94 & 54 & 134\\ 25 & 32 & 20 & 48\end{pmatrix} \sim \begin{pmatrix}25 & 31 & 17 & 43\\ 0 & 1 & 2 & 3\\ 0 & 1 & 3 & 5\\ 0 & 1 & 3 & 5\end{pmatrix} \sim \begin{pmatrix}25 & 31 & 17 & 43\\ 0 & 1 & 2 & 3\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0\end{pmatrix}$$

bu matritsaning rangi $\begin{pmatrix}31 & 17 & 43\\ 1 & 2 & 3\\ 0 & 1 & 2\end{pmatrix}$ matritsa rangiga teng.

$\begin{vmatrix}31 & 17 & 43\\ 1 & 2 & 3\\ 0 & 1 & 2\end{vmatrix}=40\neq 0$ $\qquad r\begin{pmatrix}31 & 17 & 43\\ 1 & 2 & 3\\ 0 & 1 & 2\end{pmatrix}=3.$

Demak, berilgan matritsaning rangi ham 3 ga teng. $r(A)=3$

(1) ko`rinishdagi $A$ matritsa uchun $det A \neq 0$ bo`lsa, teskari matritsa 2 usulda topiladi:

1. Klassik usuli;
2. Jordan usuli.

Misol 3. $A=\begin{pmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 1 & -2 & -1 \end{pmatrix}$ matritsa uchun teskari A^{-1} matritsani klassik usulda toping.

Klassik usulda teskari matritsa $A^{-1}=\frac{1}{|A|}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$ (2)

formula bo'yicha hisoblanadi. Bu yerda $|A|$ berilgan matritsa determinanti. A_{ij}(i=1, 2, 3; j=1, 2, 3) berilgan matritsaning algebraik to'ldiruvchilari.

$|A|=\begin{vmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 1 & -2 & -1 \end{vmatrix}=-2+12-20-2+15+16=43-24=19\neq 0.$ Demak, A matritsa maxsus emas matritsa. A^{-1} teskari matritsa mavjud. Algebraik to'ldiruvchilarini hisoblaymiz:

$A_{11}=\begin{vmatrix} 1 & 4 \\ -2 & -1 \end{vmatrix}=-1+8=7$ $A_{21}=-\begin{vmatrix} 3 & 2 \\ -2 & -1 \end{vmatrix}=-(-3+4)=-1$

$A_{31}=\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}=12-2=10$ $A_{12}=-\begin{vmatrix} 5 & 4 \\ 1 & -1 \end{vmatrix}=-(-5-4)=9$

$A_{22}=\begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix}=-2-2=-4$ $A_{32}=-\begin{vmatrix} 2 & 2 \\ 5 & 4 \end{vmatrix}=-(8-10)=2$

$A_{13}=\begin{vmatrix} 5 & 1 \\ 1 & -2 \end{vmatrix}=-10-1=-11$ $A_{23}=-\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix}=-(-4-3)=7$

$A_{33}=\begin{vmatrix} 2 & 3 \\ 5 & 1 \end{vmatrix}=2-15=-13$

A_{ij} larni (2) formulaga qo'yamiz:

$A^{-1}=1/19\begin{pmatrix} 7 & -1 & 10 \\ 9 & -4 & 2 \\ -11 & 7 & -13 \end{pmatrix}$ teskari matritsaning to'g'ri topilganini

$AA^{-1}=E$ (3)

formula bo'yicha tekshiramiz:

$$\begin{pmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 1 & -2 & -1 \end{pmatrix}*1/19\begin{pmatrix} 7 & -1 & 10 \\ 9 & -4 & 2 \\ -11 & 7 & -13 \end{pmatrix}=1/19*\begin{pmatrix} 14+27-22 & -2-12+14 & 20+6-26 \\ 35+9-44 & -5-4+28 & 50+2-52 \\ 7-18+11 & -1+8-7 & 10-4+13 \end{pmatrix}=$$

$$=1/19*\begin{pmatrix}19 & 0 & 0\\ 0 & 19 & 0\\ 0 & 0 & 19\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=E$$

Demak, A^{-1} to`g`ri topilgan.

Misol 4. $A=\begin{pmatrix}1 & 2 & 1\\ -1 & -1 & -3\\ 4 & 3 & -2\end{pmatrix}$

$|A|=16\neq 0$ teskari matritsa mavjud. Teskari matritsani Jordan usulida topamiz. Berilgan matritsani birlik matritsa hisobida kengaytirib, faqat satrlar ustida elementar almashtirishlar bajaramiz, bu usulni to chap tomonda A matritsa o'rnida birlik matritsa hosil bo'lguncha davom ettiramiz, o'ng tomonda hosil bo'lgan matritsa berilgan matritsaga nisbatan teskari matritsa bo'ladi.

$(A|E)\sim(E|A^{-1})$ - Jordan usuli algoritmi.

$$\left(\begin{array}{ccc:ccc}1 & 2 & 1 & 1 & 0 & 0\\ -1 & -1 & -3 & 0 & 1 & 0\\ 4 & 3 & -2 & 0 & 0 & 1\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1 & 2 & 1 & 1 & 0 & 0\\ 0 & 1 & -2 & 1 & 1 & 0\\ 0 & -5 & -6 & -4 & 0 & 1\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1 & 2 & 1 & 1 & 0 & 0\\ 0 & 1 & -2 & 1 & 1 & 0\\ 0 & 0 & -16 & 1 & 5 & 1\end{array}\right)$$

$$\sim\left(\begin{array}{ccc|ccc}1 & 2 & 1 & 1 & 0 & 0\\ 0 & 1 & -2 & 1 & 1 & 0\\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1 & 0 & 5 & -1 & -2 & 0\\ 0 & 1 & 0 & 14/16 & 6/16 & -2/16\\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16\end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccc|ccc}1 & 0 & 0 & -11/16 & -7/16 & 5/16\\ 0 & 1 & 0 & 14/16 & 6/16 & -2/16\\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16\end{array}\right)\qquad A^{-1}=1/16\begin{pmatrix}-11 & -7 & 5\\ 14 & 6 & -2\\ -1 & -5 & -1\end{pmatrix}$$

teskari matritsa to'g'ri topilganini (3) formulaga qo'yib, tekshiramiz:

$$AA^{-1}=1/16\begin{pmatrix}1 & 2 & 1\\ -1 & -1 & -3\\ 4 & 3 & -2\end{pmatrix}*\begin{pmatrix}-11 & -7 & -5\\ 14 & 6 & -2\\ -1 & -5 & -1\end{pmatrix}=$$

$$=1/16\begin{pmatrix}-11+28-1 & -7+12-5 & 5-4-1\\ 11-14+3 & 7-6+15 & -5+2+3\\ -44+42+2 & -28+18+10 & 20-6+2\end{pmatrix}=$$

$$=1/16\begin{pmatrix}16 & 0 & 0\\ 0 & 16 & 0\\ 0 & 0 & 16\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$ demak, teskari matritsa to'gri topilgan.

Misol 5. Matritsa normasini toping: $A = \begin{pmatrix} 2 & 4 & 1 \\ 3 & 5 & 7 \end{pmatrix}$

$$N(A) = \sqrt{2^2 + 3^2 + 4^2 + 5^2 + 1^2 + 7^2} = \sqrt{104}$$

Mustaqil yechish uchun misollar

Berilgan kvadrat matritsaning determinantlari, normalari va ranglarini topilsin:

4.1. a) $A=\begin{pmatrix} 1 & 2 \\ -2 & 0 \end{pmatrix}$ b) $A=\begin{pmatrix} -1 & 0 & 8 \\ 5 & 9 & 0 \\ 0 & 4 & 3 \end{pmatrix}$

c) $A=\begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 2 \end{pmatrix}$ d) $A=\begin{pmatrix} 2 & 3 & 4 & 0 \\ 1 & 5 & 7 & 0 \\ 3 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Quyidagi matritsalar rangini minorlar ajratish usuli bilan hisoblang:

4.2. $A=\begin{pmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{pmatrix}$ 4.3. $A=\begin{pmatrix} 2 & 1 & -2 & 3 \\ -2 & 9 & -4 & 7 \\ -4 & 3 & 1 & -1 \end{pmatrix}$

4.4. $A=\begin{pmatrix} 3 & 5 & 7 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}$ 4.5. $A=\begin{pmatrix} 0 & 2 & -4 \\ -1 & -4 & 5 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \\ 2 & 3 & 0 \end{pmatrix}$

4.6. $A=\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}$ 4.7. $A=\begin{pmatrix} 1 & 2 & 1 & 3 \\ 4 & -1 & -5 & -6 \\ 1 & -3 & -4 & -7 \\ 2 & 1 & -1 & 0 \end{pmatrix}$

Quyidagi matritsalar rangini elementar almashtirish usuli bilan hisoblang:

4.8. $\begin{pmatrix} 1 & 2 & 1 & 3 & 4 \\ 3 & 4 & 2 & 6 & 8 \\ 1 & 2 & 1 & 8 & 4 \end{pmatrix}$ 4.9. $\begin{pmatrix} 1 & 7 & 5 & 8 & 9 & 2 \\ 3 & 21 & 15 & 24 & 27 & 6 \\ 2 & 14 & 10 & 16 & 18 & 4 \end{pmatrix}$

4.10. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix}$ 4.11. $\begin{pmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 2 & 0 & 4 & 0 & 0 \end{pmatrix}$

4.12. $\begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 8 & 6 & -7 & 4 & 2 \\ 4 & 3 & -8 & 2 & 7 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & -6 \end{pmatrix}$ 4.13. $\begin{pmatrix} 24 & 19 & 36 & 72 & -38 \\ 49 & 40 & 73 & 147 & -80 \\ 73 & 59 & 98 & 219 & -118 \\ 47 & 36 & 71 & 141 & -72 \end{pmatrix}$

4.14. $\begin{pmatrix} 17 & -28 & 45 & 11 & 39 \\ 24 & -37 & 61 & 13 & 50 \\ 25 & -7 & 32 & -18 & -11 \\ 31 & 12 & 19 & -43 & -55 \\ 42 & 13 & 29 & -55 & -68 \end{pmatrix}$ 4.15. $\begin{pmatrix} 47 & -67 & 35 & 201 & 155 \\ 26 & 98 & 23 & -294 & 6 \\ 16 & -428 & 1 & 1284 & 52 \end{pmatrix}$

Berilgan kvadrat matritsalar uchun teskari matritsani ikki usulda toping:

4.16. $\begin{pmatrix} -1 & 1 \\ 4 & 2 \end{pmatrix}$ 4.17. $\begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$

4.18. $\begin{pmatrix} tga & 1 \\ 2 & ctga \end{pmatrix}$ 4.19. $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 2 \end{pmatrix}$

4.20. $\begin{pmatrix} 1 & -1 & 1 \\ -38 & 41 & -34 \\ 27 & -29 & 24 \end{pmatrix}$ 4.21. $\begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}$

4.22. $\begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix}$

Quyidagi matritsali tenglamalarni eching:

4.23. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} X = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix}$ 4.24. $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} X = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}$

Berilgan matritsalarning determinanti, normasi va rangi topilsin:

4.25. a) $A = \begin{pmatrix} 2 & 5 \\ -4 & 2 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 0 & 5 \\ 4 & -2 & -1 \\ 2 & 1 & 3 \end{pmatrix}$

c) $A=\begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 2 \end{pmatrix}$ d) $A=\begin{pmatrix} 1 & 2 & 1 & 0 \\ 1 & 1 & 3 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 0 \end{pmatrix}$

Matritsalarning ranglari topilsin:

4.26. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix}$ 4.27. $\begin{pmatrix} 1 & 2 & 3 & 6 \\ 2 & 3 & 1 & 6 \\ 3 & 1 & 2 & 6 \end{pmatrix}$

4.28. $\begin{pmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & 3 & 0 \end{pmatrix}$ 4.29. $\begin{pmatrix} 4 & 5 & 2 & 1 & -3 \\ 0 & 2 & 1 & 1 & 2 \\ 4 & 7 & 3 & 3 & -1 \\ 8 & 12 & 5 & 3 & -4 \end{pmatrix}$

4.30. $\begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{pmatrix}$ 4.31. $\begin{pmatrix} 3 & -1 & 3 & 2 & 5 \\ 5 & -3 & 2 & 3 & 4 \\ 1 & -3 & -5 & 0 & -7 \\ 7 & -5 & 1 & 4 & 1 \end{pmatrix}$

Matritsaning teskarisini toping:

4.32. $\begin{pmatrix} 1 & 5 & 7 \\ 3 & 1 & 1 \\ 2 & 3 & 4 \end{pmatrix}$ 4.33. $\begin{pmatrix} 2 & -1 & 7 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{pmatrix}$

4.34. $\begin{pmatrix} 1 & 0 & -2 \\ 3 & 1 & 0 \\ -1 & 2 & 4 \end{pmatrix}$ 4.35. $\begin{pmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{pmatrix}$

Quyidagi matritsali tenglamani eching:

4.36. $\begin{pmatrix} 1 & -3 \\ 4 & -6 \end{pmatrix} X = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$