O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

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20. SONLI KETMA-KETLIKLARNING LIMITI

Har bir n natural songa aniq bir x_n haqiqiy sonni mos qo‘yuvchi qonun berilgan bo‘lsa, R_1 haqiqiy sonlar o‘qida $x_1, x_2, ..., x_n, ...,$ yoki $\{x_n\}$ *nuqtalar (sonlar) ketma-ketligi* berilgan deyiladi.

Masalan, har bir n natural songa $x_n=\frac{3(n+1)}{2n}$ son mos qo‘yilgan bo‘lsa,

$$3;\ \frac{9}{4};\ 2;\ \frac{15}{8};\ \ldots;\ \frac{3(n+1)}{2n};\ \ldots$$

sonlar ketma-ketligi berilganligini anglatadi.

a sonning har qanday oldindan tayinlangan ε atrofi uchun $\{x_n\}$ sonli ketma-ketlikning shunday bir N tartib raqamini (ε ga bog‘liq ravishda) tanlash mumkin bo‘lsak, barcha $n>N$ tartib raqamli hadlari uchun $|x_n - a| < \varepsilon$ tengsizlikni qanoatlantirsa, a soni $\{x_n\}$ *sonli ketma-ketlikning limiti* deyiladi.

n o‘lchovli haqiqiy fazoda har bir k natural songa aniq bir n o‘lchovli M_k nuqtani mos qo‘yuvchi qonuniyat o‘rnatilgan bo‘lsa, R_n fazoda cheksiz *n o‘lchovli nuqtalarning ketma-ketligi* berilgan deyiladi va $M_1, M_2, ..., M_k, ...,$ yoki $\{M_k\}$ ko‘rinishda yoziladi.

Masalan, har bir k natural songa ikki o‘lchovli $M_k\ (2k;\ \frac{3}{k})$ nuqta mos qo‘yilgan bo‘lsin. Bu esa, R_2 haqiqiy koordinatalar tekisligida

$$M_1(2;\ 3),\ M_2(4;\ \frac{3}{2}),\ M_3(6;\ 1),\ ...,\ M_n(2k;\ \frac{3}{k}),\ ...$$

nuqtalar ketma-ketligi berilganligini anglatadi.

n o‘lchovli M_0 nuqtaning har qanday ε atrofida berilgan nuqtalar ketma-ketligining biror-bir mos tartib raqamidan boshlab, barcha hadlari tegishli bo‘lsa, ya‘ni har qanday oldindan tayinlanadigan ε >0 uchun K tartib raqamni (ε ga bog‘liq ravishda) ko‘rsatish mumkin bo‘lsaki, barcha $k>K$ tartib raqamli hadlar $M_k \in S_\varepsilon(M_0)$ bo‘lsa, M_0 nuqtaga $\{M_k\}$ *nuqtalar ketma-ketligining limiti* deyiladi va $\lim\limits_{k\to\infty}\{M_k\} = M_0$ ko‘rinishida yoziladi.

Mustaqil yechish uchun misollar

20.1. Umumiy hadi orqali berilgan ketma-ketlikning birinchi beshta hadini yozing:

a) $x_n = \frac{1}{2n+1}$ b) $x_n = \frac{n+2}{n^3+1}$

c) $x_n = \frac{n}{2n+1}$ d) $x_n = (-1)^n \frac{n+1}{n^2}$.

20.2. Ketma-ketlikning berilgan hadlari orqali umumiy hadining formulasini yozing:

a) $1; \frac{1}{2}; \frac{1}{6}; \frac{1}{24}; \ldots$ b) $1;\ 2\frac{1}{4};\ 2\frac{7}{9};\ 3\frac{1}{16};\ 3\frac{6}{25}; \ldots$

c) 2; 10; 26; 82; 242; 730; …

20.3. Sonli ketma-ketlik chegaralanganligini isbotlang:

a) $x_n = \frac{n^2+1}{n^2+2}$

Isbot: $\frac{n^2+1}{n^2+2} = 1 - \frac{1}{n^2+2}$ va $0 < \frac{1}{n^2+2} \le \frac{1}{2}$ shuning uchun $\frac{1}{2} < x_n < 1$.

b) $x_n = \frac{(-1)^n\ n+1}{\sqrt{n^2+2}}$ c) $x_n = Sin\ n$

d) $x_n = (1-(-1)^n)$

20.4. Sonli ketma-ketlik monotonligini isbotlang:

a) $x_n = \lg n - \lg(n-1),\ (n>1)$

Isbot: $x_n = \lg n - \lg(n-1) = \lg\frac{n}{n-1}$

$x_{n+1} - x_n = \lg\frac{n+1}{n} - \lg\frac{n}{n-1} = \lg\frac{n^2-1}{n^2} = \lg(1-\frac{1}{n^2}) < 0$. Demak,

$x_{n+1} - x_n < 0,\ \ x_{n+1} < x_n$, shuning uchun bu ketma-ketlik monoton kamayuvchi.

b) $x_n = 3^n - 2^n$ c) $x_n = \sqrt{n^2-1}$

d) $x_n = \sum_{k=1}^{n} k$.

20.5. Ketma-ketlik limiti ta'rifidan foydalanib, quyidagilarni isbotlang:

a) $\lim\limits_{n\to\infty}\frac{n}{n+1}=1$

Isbot: ixtiyoriy $\varepsilon>0$ son olamiz, $|x_n-1|=\left|\frac{n}{n+1}-1\right|=\frac{1}{n+1}$; $|x_n-1|<\varepsilon$ tengsizlikni qanoatlantiruvchi *n* larni topish uchun $\frac{1}{n+1}\prec\varepsilon$ tengsizlikni yechamiz. $n>\frac{1-\varepsilon}{\varepsilon}$. Shunday qilib, $\frac{1-\varepsilon}{\varepsilon}$ sonining butun qismi $N=\left[\frac{1-\varepsilon}{\varepsilon}\right]$ bo‘ladi, u holda $|x_n-1|<\varepsilon$ tengsizlik barcha $n>N$ larda bajariladi. ε-ixtiyoriy son bo‘lgani uchun $\lim\limits_{n\to\infty}\frac{n}{n+1}=1.$

Agar $\varepsilon=0.01$ bo‘lsa, $N=\left[\frac{1-0.01}{0.01}\right]=99,\ n>99$ larda $|x_n-1|<0.01$ bo‘ladi.

b) $\lim\limits_{n\to\infty}\frac{4n-1}{2n+1}=2$ c) $\lim\limits_{n\to\infty}\frac{3n+1}{5n-1}=\frac{3}{5}$

d) $\lim\limits_{n\to\infty}\frac{2n-1}{2-3n}=-\frac{2}{3}$ qaysi *n* dan boshlab, $\left|\frac{2n-1}{2-3n}-\left(-\frac{2}{3}\right)\right|<0.0001$ tengsizlik o‘rinli bo‘ladi?

Quyidagi limitlarni toping:

20.6. $\lim\limits_{n\to\infty}\frac{3n^3+2}{4n^3-1}$ 20.7. $\lim\limits_{n\to\infty}\frac{2n^3+3}{n^3+n-1}$

20.8. $\lim\limits_{n\to\infty}\frac{(n+1)^3}{5n^3+1}$ 20.9. $\lim\limits_{n\to\infty}\frac{\sqrt[3]{n^2+n}}{4+n}$

20.10. $\lim\limits_{n\to\infty}\frac{\sqrt{2n^4+n^2+1}}{2n+n^2-1}$ 20.11. $\lim\limits_{n\to\infty}\frac{n!}{(n+1)!-n!}$

20.12. $\lim\limits_{n\to\infty}\frac{1+\frac{1}{2}+\ldots+\frac{1}{2^n}}{1+\frac{1}{3}+\ldots+\frac{1}{3^n}}$ 20.13. $\lim\limits_{n\to\infty}\frac{3+6+9+\ldots+3n}{n^2+4}$

20.14. $\lim\limits_{n\to\infty}\sqrt{n}(\sqrt{n+2}-\sqrt{n-3})$ 20.15. $\lim\limits_{n\to\infty}\frac{(2n+1)!+(2n+2)!}{(2n+3)!-(2n+2)!}$

20.16. $\lim\limits_{n\to\infty}\sqrt{n^3+8}(\sqrt{n^3+2}-\sqrt{n^3-1})$ 20.17. $\lim\limits_{n\to\infty}\left(\frac{1}{3}+\frac{1}{15}+\ldots+\frac{1}{4n^2-1}\right)$

20.18. $\lim\limits_{n\to\infty}(\frac{1}{2n}Cos\,n^3-\frac{3n}{6n+1})$ 20.19. $\lim\limits_{n\to\infty}\left(\frac{n}{n^2+1}Sin\,n!+\frac{2n^2}{1-9n^2}\right)$

20.20. R^2 fazoda quyidagi ketma-ketliklarning limiti $a \in R^2$ ekanligini isbotlang:

a) $\{x^{(n)}\} = \left\{\frac{1}{n}, \frac{1}{n}\right\}$ $a = (0,0)$, $\forall \varepsilon > 0$ son olaylik.

$$\rho(x^n, a) = \rho\left(\left(\frac{1}{n}, \frac{1}{n}\right), (0,0)\right) = \sqrt{\left(\frac{1}{n} - 0\right)^2 + \left(\frac{1}{n} - 0\right)^2} = \sqrt{\frac{2}{n^2}} < \varepsilon, \quad n > \frac{\sqrt{2}}{\varepsilon}; \ N = \left[\frac{\sqrt{2}}{\varepsilon}\right]$$

bo‘ladi, u holda $\rho(x^{(n)}, a) < \varepsilon$ tengsizlik barcha $n > N$ larda bajariladi. Ta’rifga ko‘ra $\lim\limits_{n\to\infty}\left(\frac{1}{n}, \frac{1}{n}\right) = (0,0) = a$.

b) $\{x^{(n)}\} = \left\{\left(\frac{3}{n}, \frac{1}{n^2}\right)\right\}$, $a = (0, 0)$

c) $\{x^{(n)}\} = \left\{\left(\frac{3n}{2n-1}, \frac{2-n}{2+n}\right)\right\}$, $a = \left(\frac{3}{2}, -1\right)$

R^2 fazoda ketma-ketliklar limitini toping:

20.21. $x^{(n)} = \left(\left(\frac{n-1}{n}\right)^5, \frac{n^3+27}{n^4-15}\right)$

20.22. $x^{(n)} = \left(\frac{9n+5}{4n}, \frac{1}{2n+4}\right)$

20.23. $x^{(n)} = \left(\frac{3n^4-2}{\sqrt{n^8+3n}}, \sqrt{n^2+8n} - \sqrt{n^2+4n}\right)$

20.24. Berilgan ketma-ketliklarning 5 ta hadini va umumiy hadi formulasini yozing:

a) $x_1 = 1; \ x_{n+1} = n!$

b) $x_1 = 1; \ x_{n+1} = x_n + 3$

20.25. Ketma-ketlik chegaralanmaganligini isbotlang:

a) $x_n = n^{(-1)^n}$

b) $x_n = (-1)^n n$

c) $x_n = n \cdot \log_{\frac{1}{2}} n$

d) $x_n = tg\ n$

20.26. Ketma-ketlik limitini ta’rifidan foydalanib isbotlang:

a) $\lim\limits_{n\to\infty} \frac{4n-1}{5n+1} = \frac{4}{5}$

b) $\lim\limits_{n\to\infty} \frac{\sqrt{n^2+n}}{n} = 1$

c) $\lim\limits_{n\to\infty} \frac{3n-1}{5n+1} = \frac{3}{5}$, qaysi n dan boshlab $\left|\frac{3n-1}{5n+1} - \frac{3}{5}\right| < 0.001$ tengsizlik o`rinli bo‘ladi?

Quyidagi limitlarni toping:

20.27. $\lim\limits_{n\to\infty}\frac{3n^3-4}{n^3+6}$

20.28. $\lim\limits_{n\to\infty}\frac{n^2+n+1}{(n+1)^2}$

20.29. $\lim\limits_{n\to\infty}\frac{(n+1)^4+(n-1)^4}{n^4+10}$

20.30. $\lim\limits_{n\to\infty}\frac{\sqrt[3]{4n^3+2n-1}}{2n+2}$

20.31. $\lim\limits_{n\to\infty}\frac{\sqrt{2n^2+1}}{\sqrt[4]{n^4+3n-1}}$

20.32. $\lim\limits_{n\to\infty}\frac{(n+1)!+n!}{(n+2)!}$

20.33. $\lim\limits_{n\to\infty}\frac{1+2+...+n}{n^2}$

20.34. $\lim\limits_{n\to\infty}\left(\frac{1}{n}Cos\frac{n\pi}{2}+1\right)$

20.35. $\lim\limits_{n\to\infty}\frac{1+3+...+(2n-1)}{n\sqrt{n^2+1}}$

20.36. $\lim\limits_{n\to\infty}\frac{\log_a(n+1)!}{\log_a n!}$ $(n>2,\ a>1)$

20.37. $\lim\limits_{n\to\infty}\frac{1+a+a^2+...+a^{n-1}}{1+b+b^2+...+b^{n-1}}$ $(b>a>0)$

20.38. $\lim\limits_{n\to\infty}\frac{(n+k)!+n!}{(n+k)!-n!}$

20.39. $\lim\limits_{n\to\infty}\sqrt{n}(\ln(n+2\sqrt{n}+1)-\ln n)$

20.40. $\lim\limits_{n\to\infty}(\sqrt{2+4+6+...+2n}-\sqrt{1+3+5+...+(2n-1)})$