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17. CHIZIQLI OPERATOR

17.1. Agar R^3da chiziqli $\tilde{A}$ operator $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda o'zining $A = \begin{bmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{bmatrix}$ matritsasi bilan berilgan bo'lsa, $\vec{x} = 4\vec{e}_1 - 3\vec{e}_2 + \vec{e}_3$ vektorning *y=A(x)* aksini toping.

Y=AX formulaga binoan, $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{bmatrix} * \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ -13 \\ -18 \end{bmatrix}$

Demak, $y = 10e_1 - 13e_2 - 18e_3$

17.2. $\vec{e}_1, \vec{e}_2$ bazisda $\tilde{A}$ operator $A = \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}$ matritsaga ega.

$e_1 = e_1 - 2e_2, \quad e_2 = 2e_2 + e_2$ bazisida $\tilde{A}$ operatorining matritsasini toping.

O'tish matritsasi $C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$ ning teskari matrisasi $C^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

Demak, $B = C^{-1}AC = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 8 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}$

17.3. Chizqli $\tilde{A}$ operator $A = \begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

Xarakteristik tenglama tuzamiz:

$$|A - \lambda E| = \begin{vmatrix} 1-\lambda & 4 \\ 9 & 1-\lambda \end{vmatrix} = 0, \quad \lambda^2 - 2\lambda - 35 = 0; \quad \lambda_1 = -5, \quad \lambda_2 = 7$$

λ_1=-5 ga tegishli $X^{(1)}=(X_1,X_2)$ xos vektorni topamiz. Buning uchun quyidagi tenglamani echamiz:

$$\lambda_1 = -5 \quad (A - \lambda E) \cdot x = 0 \qquad \begin{pmatrix} 6 & 4 \\ 9 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x_2 = -1{,}5x_1$$

agar $x_1=C$ deb olsak $x_2=-1.5C$, $X^{(1)}=(C;-1.5C)$ vektorlar har qanday $C \neq 0$ uchun *A* operatorni xos qiymati λ_1=-5 ga tegishli xos vector bo'ladi. Xuddi shunday

λ_2=7 xos qiymati uchun *A* operatorni xos vektorlarni $X^{(2)}=\left(\frac{2}{3}C_1, C_1\right)$, $C_1\neq 0$ vektorlar tashkil etadi.

17.4. Chiziqli operatorning $A=\begin{pmatrix}1 & 4\\ 9 & 1\end{pmatrix}$ matritsasini diogonal ko'rinishiga keltiring.

$A=\begin{pmatrix}1 & 4\\ 9 & 1\end{pmatrix}$ matrisa bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarning 3-misolda topilgan: λ_1=-5 λ_2=7

$X^{(1)}=(C;-1{,}5C)$; $X^{(2)}=\left(\frac{2}{3}C_1, C_1\right)$; $X^{(1)}$ *va* $X^{(2)}$ vektorning koordinatalari proportsional emas, shuning uchun $X^{(1)}$ va $X^{(2)}$ vektorlar chiziqli erkli. Demak, $X^{(1)}$ va $X^{(2)}$ bazisda *A*-matritsaning diagonal ko'rinishi:

$A^*=\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}$ *ёки* $A^*=\begin{pmatrix}-5 & 0\\ 0 & 7\end{pmatrix}$. Buni tekshirish uchun bazis vektorlar sifatida $X^{(1)}$=(2; -3), $X^{(2)}$=(4; 6) vektorlarni olsak, yangi bazisga o'tkazuvchi o'tish matritsa *C* ning ko'rinishi: $\begin{pmatrix}2 & 4\\ -3 & 6\end{pmatrix}$ bo'ladi. Dioganal matrtisa:

$$A^*=C^{-1}AC=\begin{pmatrix}2 & 4\\ -3 & 6\end{pmatrix}^{-1}\begin{pmatrix}1 & 4\\ 9 & 1\end{pmatrix}\begin{pmatrix}2 & 4\\ -3 & 6\end{pmatrix}=\frac{1}{24}\begin{pmatrix}6 & -4\\ 3 & 2\end{pmatrix}\begin{pmatrix}1 & 4\\ 9 & 1\end{pmatrix}\begin{pmatrix}2 & 4\\ -3 & 6\end{pmatrix}=\frac{1}{24}\begin{pmatrix}-30 & 20\\ 21 & 14\end{pmatrix}$$

$$\begin{pmatrix}2 & 4\\ -3 & 6\end{pmatrix}=\frac{1}{24}\begin{pmatrix}-120 & 0\\ 0 & 168\end{pmatrix}=\begin{pmatrix}-5 & 0\\ 0 & 7\end{pmatrix}$$

Mustaqil yechish uchun misollar

17.5. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli $\tilde{A}$ operator $A=\begin{pmatrix}3 & 2\\ -1 & 5\end{pmatrix}$ matritsa bilan berilgan, $x=4e_1-3e_2$ bo'lsa, $y=A(x)$ ni toping.

17.6. $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator $\begin{pmatrix}-1 & 0 & 2\\ 2 & 1 & -7\\ 3 & 0 & 1\end{pmatrix}$ matritsa bilan berilgan $x=2e_1-4e_2-e_3$ bo'lsa, $y=A(x)$ ni toping.

17.7. $\vec{e}_1, \vec{e}_2$ bazisda $\tilde{A}$ operator $A = \begin{pmatrix} 2 & 4 \\ -3 & 3 \end{pmatrix}$ matritsaga ega.

$e_1' = e_2 - 2e_1, \quad e_2' = 2e_1 - 4e_2$ bazisda $\tilde{A}$ operatorning matritsasini toping.

Berilgan matritsalarning xos qiymatlari va xos vektorlarini toping:

17.8. $A = \begin{pmatrix} 2 & 4 \\ -1 & -3 \end{pmatrix}$

17.9. $A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ 1 & 3 & 0 \end{pmatrix}$

17.10. $A = \begin{pmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{pmatrix}$

17.11. $\vec{e}_1, \vec{e}_2, \vec{e}_3$, bazisdan $\vec{e}_2, \vec{e}_3, \vec{e}_1$ bazisga o'tish matrisasini toping.

17.12. $\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4$ bazisda $\tilde{A}$ operatorining matritsasi

$A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 3 & -1 & 2 \\ 2 & 5 & 0 & 1 \\ 1 & 2 & 1 & 3 \end{pmatrix}$ berilgan. Ushbu operatorning

1) $\vec{e}_1, \vec{e}_3, \vec{e}_2, \vec{e}_4$ bazisdagi matritsasini toping;

2) $e_1;\ e_1 + e_2;\ e_1 + e_2 + e_3;\ e_1 + e_2 + e_3 + e_4$ bazisdagi matritsasini toping.

O'zlarining matritsalari bilan berilgan chiziqli operatorlarning xos qiymatlari va xos vektorlarini toping:

17.13. $A = \begin{pmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{pmatrix}$;

17.14. $A = \begin{pmatrix} 4 & -5 & 2 \\ 5 & -7 & 3 \\ 6 & -9 & 4 \end{pmatrix}$;

17.15. $A = \begin{pmatrix} 1 & -3 & 3 \\ 2 & 6 & 3 \\ -1 & -4 & 8 \end{pmatrix}$;

17.16. $A = \begin{pmatrix} 7 & -12 & 6 \\ 10 & -19 & 10 \\ 12 & -24 & 13 \end{pmatrix}$;

17.17. $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}$

Chiziqli operatorning $\tilde{A}$ matritsasini diogonal ko'rinishiga keltiring:

17.18. $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 0 \\ -2 & -2 & 1 \end{pmatrix}$

17.19. $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -6 & 1 & 7 & -1 \end{pmatrix}$