

**O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS  
TA‘LIM VAZIRLIGI**

**TOSHKENT MOLIYA INSTITUTI**

**«МАТЕМАТИКА» КАФЕДРАСИ**

**«OLIV MATEMATIKA»  
FANIDAN**

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## 28. TO‘LA DIFFERENSIALGA DOIR MISOLLAR YECHISH

Ikki, uch va undan ko‘p o‘zgaruvchiga bog‘liq bo‘lgan funksiyalar ko‘p o‘zgaruvchiga bog‘liq funksiyalar deyiladi va

$$z=f(x;y)$$

$$u=f(x;y;z) \dots$$

$$v=f(x;y;z; \dots;t)$$

kabi yoziladi. Ko‘p o‘zgaruvchiga bog‘liq funksialardan bir o‘zgaruvchisi bo‘yicha xususiy hosila hisoblaganda boshqa o‘zgaruvchilari o‘zgarmas deb qaraladi. Ko‘p o‘zgaruvchiga bog‘liq funksiyaning to‘la differensialni quyidagi formula bo‘yicha topiladi:

$z=f(x,y)$  funksiya uchun

$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

$u=f(x,y,z)$  funksiya uchun

$$du = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

28.1.  $z(x,y) = y\sqrt{x} + \frac{x}{\sqrt{y}}$  funksiya xususiy hosilalarini toping.  $z(x,y)$

funksiyadan  $x$  bo‘yicha xususiy hosilani hisoblaganda o‘zgaruvchi  $y$  ni o‘zgarmas son deb qaraymiz:

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{x}} y + \frac{1}{\sqrt{y}};$$

$z(x,y)$  funksiya o‘zgaruvchi  $y$  bo‘yicha xususiy hosilani hisoblaganda o‘zgaruvchi  $x$  ni o‘zgarmas son deb qaraymiz:

$$\frac{\partial z}{\partial y} = \sqrt{x} - \frac{x}{2y \cdot \sqrt{y}}$$

28.2.  $z(x,y) = \ln(x + \sqrt{x^2 + y^2})$  funksiyaning to‘la differensialini toping:

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

$$\frac{\partial z}{\partial x} = \frac{\left(x + \sqrt{x^2 + y^2}\right)_x}{x + \sqrt{x^2 + y^2}} = \frac{1 + \frac{(x^2 + y^2)_x}{2\sqrt{x^2 + y^2}}}{x + \sqrt{x^2 + y^2}} = \frac{1 + \frac{x}{\sqrt{x^2 + y^2}}}{x + \sqrt{x^2 + y^2}} = \frac{1}{\sqrt{x^2 + y^2}};$$

$$\frac{\partial z}{\partial y} = \frac{\left(x + \sqrt{x^2 + y^2}\right)_y}{x + \sqrt{x^2 + y^2}} = \frac{\frac{2y}{2\sqrt{x^2 + y^2}}}{x + \sqrt{x^2 + y^2}} = \frac{y}{\sqrt{x^2 + y^2} \left(x + \sqrt{x^2 + y^2}\right)}$$

$$dz = \frac{dx}{\sqrt{x^2 + y^2}} + \frac{dz}{\sqrt{x^2 + y^2} \left(x + \sqrt{x^2 + y^2}\right)};$$

### Mustaqil yechish uchun misollar

Quyidagi funksiyalarning xususiy hosilalarini toping:

$$28.3. z(x,y) = \frac{x-y}{x+y}$$

$$28.4. u = e^{\frac{x}{y}} + e^{\frac{z}{y}}$$

$$28.5. z(x,y) = -\frac{\text{Cos}x}{\text{Cos}y}$$

$$28.6. z(x,y) = \ln(x^2 - y^2);$$

$$28.7. z(x,y) = x \text{Sin}(x+y);$$

$$28.8. z(x,y) = \frac{x}{\sqrt{x^2 + y^2}}$$

$$28.9. z(x,y) = \arcsin \frac{y}{x}$$

$$28.10. u = 2y\sqrt{x} + 3y^2 \sqrt[3]{z^2};$$

$$28.11. z(x,y) = \sqrt{xy + \frac{x}{y}}$$

$$28.12. z(x,y) = e^{\frac{\sin y}{x}};$$

Quyidagi funksiyalarning to'la differensiallarini toping:

$$28.13. z = 5x^3 y^2;$$

$$28.14. z = \frac{y}{x} - \frac{x}{y};$$

$$28.15. z = (\sin x)^{\cos y};$$

$$28.16. z = e^{x^2 + y^2};$$

$$28.17. z = \arctg \frac{x}{y};$$

$$28.18. z = \sin^2 x + \cos^2 y;$$

$$28.19. z = x \ln \frac{y}{x};$$

$$28.20. z = x^y;$$

$$28.21. z = \text{tg} \frac{y}{x} + \text{ctg} \frac{x}{y};$$

$$28.22. z = \text{tg}(2x + \sqrt{y});$$

$$28.23. z = e^{xy}.$$

28.24.  $z = \sqrt{x} \sin \frac{y}{x}$ ;  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \frac{z}{2}$  ekani isbot qilinsin.

28.25.  $z = \ln(\sqrt{x} + \sqrt{y})$ ;  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \frac{1}{2}$  ekani isbot qilinsin.

Quyidagi funksiyalarning xususiy hosilalarini toping:

28.26.  $z = \cos(ax - by)$ ;      28.27.  $z = \frac{x}{3y-2x}$ ;

28.28.  $z = \ln \sin(x - 2y)$ ;      28.29.  $z = 2\cos^2\left(x - \frac{y}{2}\right)$ ;

28.30.  $z = \ln \operatorname{tg} \frac{y}{x}$ ;      28.31.  $z = \operatorname{arctg} \sqrt{xy}$ ;

28.32.  $z = y e^{\frac{x}{y}}$ ;      28.33.  $u = (x-y)(x-z)(y-z)$ .

Quyidagi funksiyalarning to'la differensialini toping:

28.34.  $z = x^m y^n$ ;      28.35.  $z = y \sqrt[3]{x}$ ;

28.36.  $z = \sqrt{x^2 + y^2}$ ;      28.37.  $z = e^{\cos(xy)}$ ;

28.38.  $z = \frac{x}{y} e^{xy}$ ;      28.39.  $u = x^{y^2 z}$ .